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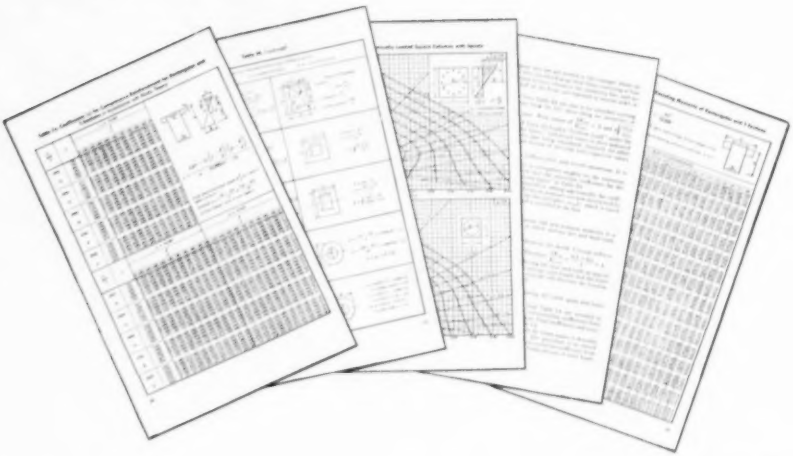

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The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

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In ACI Journal next month

SUBCOMMITTEE II of ACI COMMITTEE 325 will present its report on "Considerations in the Selection of Slab Dimensions" in the November JOURNAL. A. T. Goldbeck heads the subcommittee which has studied slab dimensions and the factors which affect them. Typical thickness design for airport pavements and highway slabs is illustrated. Significant trends in highway slab construction are noted—reduction in use of transverse expansion joints; a decided trend toward use of wider pavements . . . 24-ft width for two lanes now is most prevalent . . . also a decided trend away from thickened edge pavements.

CHARLES S. WHITNEY and EDWARD COHEN have prepared a "Guide for Ultimate Strength Design of Reinforced Concrete," intended to serve as a supplement to the ACI Building Code (ACI 318-56) which permits design of reinforced concrete members by the ultimate strength method. The method is presented in its simplest form, with working equations and charts to aid in their application.

More about lightweight concrete, third part of a series based on the work of ACI Committee 213, is the C. C. CARLSON paper "Lightweight Aggregates for Concrete Masonry Units." Differences between lightweight and dense aggregates having influence in block manufacturing procedure are cited, and the means of accommodating them are given. Mention is made of lightweight-aggregate masonry units for precast and cast-in-place floor and roof construction.

"Insulating Concretes" is the contribution of R. C. VALORE, and is the concluding part of the lightweight-concrete series. Density, strength, and thermal conductivity are compared for lightweight concretes made with perlite, vermiculite, pumice, expanded shale, and expanded slag. Cellular concretes made with sand, and without aggregate, are included in the comparisons.

A. M. LOUNT reports on design and construction in "Prestressed Bridge Supports Cranes at Niagara River Weir." The bridge is based on a homogeneous grid system of longitudinal girders tied together by diaphragms, covered with a 7-in. slab acting compositely with the longitudinal girders. All of the grid components are prestressed. Step-by-step method for exact structural analysis of the grid is summarized.

"Particle Interference in Concrete Mixes," by B. J. BUTCHER and H. J. HOPKINS, advances a new method of applying the Weymouth criterion for particle interference in concrete mixes, based on a consideration of the complete grading of solids instead of applying the criterion in turn to each successive group. Authors indicate that their examination of Weymouth's original method of calculation has shown his results to be in conflict with recognized physical laws.

Title No. 53-18

Design Considerations for Concrete Pavement Reinforcement for Crack Control*

Reported by Subcommittee IV, ACI Committee 325

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SYNOPSIS

Considerations pertinent to the design of distributed reinforcement for crack control have been suggested. Attention has been devoted to characteristics of subgrade frictional resistance, and to the relation between steel stress and crack width.

A rational method for determining coefficient of subgrade friction is advanced which follows conventional lines.

INTRODUCTION

The assignment of Committee 325 is to develop a recommended practice for the structural design of concrete highway and airport pavement (subbase, slab dimensions, joints, and reinforcement) on the basis of the best information available, and to keep such recommendations up to date as new information indicates need for revision. The development of recommended practices for the design and use of steel reinforcement in pavements is assigned to Subcommittee IV.

Published design information is available only for distributed reinforcement for crack control. This subcommittee report is, therefore, limited to design and use of steel in that classification. Published design information is based on the "subgrade drag" theory stated in Eq. (1) of this report.

In applying this theory to design over the past 30 years, a value of 1.5 has commonly been assigned to the coefficient of subgrade friction. Design stress in the steel has ranged from 50 to 100 percent of the yield strength. The use of a high value for the coefficient representing the resistance of the subgrade to horizontal

*Prepared as a part of the work of ACI Committee 325, Structural Design of Concrete Pavements for Highways and Airports. The report was submitted to the main committee (17 members) with 14 voting affirmatively and three negatively.

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†Served as chairman of Subcommittee IV, Committee 325, from February, 1949, to February, 1955.

slab movement probably compensates for the effects of other unknown forces and conditions active in the pavement. Experience indicates that application of this design method with these limitations has resulted in use of steel reinforcement with generally satisfactory performance.

Considerations given in this report attempt to take into account the variable nature of subgrade friction and steel stress. Coefficient values for subgrade friction related to subgrade character and extent of slab movements, and of steel stresses related to both strength and bond behavior, as advanced in this report, are in principle more realistic. However, as shown by appended information, available research data on the subject are inadequate. Additional information is needed before friction coefficient values, resulting in more economical reinforcement design than now in use, can be proposed with confidence. Therefore, it is the purpose of this report to present at this time design considerations rather than definite recommendations, to stimulate discussion and encourage continued research on various phases of the problem.

CRACK CONTROL

Reinforced concrete pavement for highways and airports consists mainly of slabs one lane wide between longitudinal joints or free edges, divided lengthwise by transverse joints 30 to 100 ft apart. The principal reinforcement is longitudinal and consists of hot-rolled bars or cold-drawn wires, lapped to extend continuously through the full length of each slab and distributed across the width of the slab, with the edge members near the pavement edge and longitudinal joint.

This reinforcement is intended to maintain each slab as an integral unit, regardless of cracking of the concrete, by tying the portions on each side together without appreciable separation at the crack, so that the slab will expand and contract due to temperature changes substantially as a unit.[1] This type of reinforcement and its function are classified as distributed reinforcement for crack control.

Scope of information

These design considerations present data and suggestions for this most common type of pavement reinforcement. It is limited to reinforcement in pavement slabs relatively free to undergo contraction and expansion throughout their length. Such slabs are not usually over 100 ft long, although data from continuously reinforced pavements indicate that principles of reinforcement for crack control apply to somewhat greater slab lengths, between 100 and 200 ft.[2] Specially oriented reinforcement, such as edge bars, corner bars, and two-way structural reinforcement used in bridge approach slabs, is not within the scope of this report. Strengthening effects of the steel as reinforcement in the concrete are neglected. Possible value of distributed steel in preventing cracks, although evidenced by crack surveys,[3] is not considered.

Some continuously reinforced pavements, consisting of slabs of great length without transverse joints with considerable weight of distributed longitudinal

steel, have been built. These have been carefully observed and have contributed to our understanding of pavement reinforcement.[2] However, requirements for steel in continuously reinforced pavements are not considered here.

Concrete pavement cracking

Since the function of distributed reinforcement in controlling cracks in pavement is being considered, some information on causes of pavement cracks is essential.[3] Cracks which by appearance and distribution can be considered cross sectional (generally transverse) are made the basis for reinforcement design.

Structural corner cracks may be caused by insufficient slab thickness or lack of subgrade support near corners of joints or cracks. Longitudinal cracks, which were common in two-lane pavements built prior to 1925, have been largely eliminated by longitudinal joints between lanes, limiting individual slab widths to dimensions which can accommodate transverse temperature warping. Some longitudinal cracks generally referred to as infiltration cracks may occur near edges and perpendicular to transverse joints. These various pavement failures may all be remedied in some degree by distributed steel reinforcement, but no basis exists at present for dimensioning steel with regard to such crack occurrence and its control.

Random pavement cracks, mainly transverse, are caused by subgrade movements and thermal and moisture variations in the concrete resulting in critical concrete tension, singly or in combination with load stresses. Subgrade movements are associated with settlement, consolidation, loss of subgrade by pumping, swelling due to moisture pick-up, and frost heaves, causing variation in subgrade level and support greater than can be accommodated by pavement flexure.

Cracks associated with abnormal subgrade movements at transverse joints can be expected to occur between a few feet and up to 10 or 12 ft from these joints in highway slabs. They are more frequent in the last loaded slab end in the direction of traffic. Reinforcement may also keep these pumping cracks closed and the subgrade failure localized near the joints. Cracks due to temperature and shrinkage generally appear with increasing frequency as slab lengths are increased.

REINFORCEMENT DESIGN PRINCIPLES

General premises

Reinforcement for crack control in practice is dimensioned to keep transverse cracks near the center of long slabs from opening into wide cracks. A transverse crack in an unreinforced pavement slab results in two independent slabs divided by an irregular opening of changing width. Any open crack in the pavement is a point for stress concentration, increased deflection, and possibly subgrade deterioration. These influences are diminished if the cracks can be prevented from open-

ing in injurious degree. Crack-control reinforcement is intended to insure the following:

1. Intimate contact of adjacent slabs to produce a degree of load transfer across the crack through aggregate interlock, so that wheel loads near the crack are supported by the slabs on both sides, with correspondingly smaller concrete flexural stress at the crack and least possible increase in pavement deflections at the crack.
2. Minimization of water seepage to the subgrade and of soil infiltration into the crack.

Design criteria

When temperature drops, the pavement slab shortens. Frictional resistance to the contraction increases with increasing distance from each slab end, or from each crack free to open and close. Friction of the subgrade must be overcome by the tension resistance of the steel crossing a crack assumed to occur at the center of the slab length. The forces increase with increasing weight, thickness, and length of slab, and with increasing frictional coefficient.

Frictional resistance to movement, from the slab end to the crack, is overcome by the steel crossing the crack.[1] The cross sectional area of steel required per foot width of slab at the crack is:

$$A_s = F \frac{Lw}{24\ f_s} \qquad \text{.....(1)}$$

L = slab length, in. ($L/2$ is distance from end to center of slab)
w = weight, lb per sq ft of slab
F = coefficient of friction between slab and subgrade. Generally assumed to be 1.5
f_s = allowable steel stress at crack, psi
A_s = cross-sectional area of steel in sq in. per ft width of pavement

This simple formula gives the area of steel required half-way between transverse joints, provided the subgrade frictional coefficient is known.

Prognostication of the coefficient of friction between slab and subgrade for a particular pavement and the selection of appropriate steel stress are necessary to apply the simple formula in pavement design. These will be dealt with separately.

Subgrade frictional resistance

Sliding friction in the normal sense does not take place between pavement slabs of conventional lengths and the subgrade. The movement is induced by temperature changes in the concrete. The subgrade is capable of considerable movement within itself and is also subject to temperature changes. The frictional resistance therefore varies with amount of movement, size of slab, rate of temperature changes, and characteristics of the subgrade. Subgrade resistance varies with presence of moisture between the slab and the soil, roughness at the boundary, and soil cohesiveness.

Available experimental research, both published and as supplied to this committee from unpublished tests, is summarized in Appendix A with tentative in-

dications. The data are meager, and correlation with pavement performance is completely lacking. Possible influence of traffic has not been investigated.

For sliding friction, a frictional coefficient of 1.5 is indicated, applicable to parts of slabs sliding over the subgrade, and this value is commonly used in design for the case of sliding and nonsliding friction. While this value undoubtedly is high, its use probably compensates for other factors which are not considered in design because of lack of knowledge about their magnitudes.

Transverse joint resistance

Reinforcement frictional stresses have been analyzed on the assumption that no resistance to contraction occurs at transverse joints. Load-transfer devices across such joints may substantially resist slip if inadequately installed, if not adequately coated to prevent bonding, or if rusted to seizure in the concrete. Bond resistance of dowels of 100 psi may not be unusual and may result in about 25 psi tension stress on a pavement cross section. Any dowel slip resistance must be considered an additional force to be overcome by crack-control reinforcement. Dowels of smooth-surfaced rust resistant alloy or dowels encased in smooth nonrusting metal sheaths, if properly aligned, and lubricated, can be assumed to offer negligible resistance to slab movements.

RATIONAL METHOD FOR DETERMINING COEFFICIENT OF FRICTION

A method developed by B. F. Friberg and explained below indicates the variation in the coefficient of friction for the case of sliding and nonsliding friction, and equations are shown which will give a more refined value for the coefficient of friction in Eq. (1), rather than using the coefficient of sliding friction for all conditions.

For daily temperature cycles, subgrade frictional resistance appears to increase from zero at very small movement substantially along a parabolic curve (and proportional to the square root of movement) until free sliding takes place, with a constant friction coefficient for parts of the slab subject to free sliding. Use of this relationship in design was proposed by E. F. Kelley[4] and credited by him to the late R. D. Bradbury, formerly engineer-director, Wire Reinforcement Institute.

The friction coefficient F_n' under any point along a slab subject to unrestrained contraction and expansion from its center, as well as the average friction under each half-slab, can be computed from a general relationship shown in Eq. (2). The variable frictional resistance related to slab movement will be termed the coefficient of nonsliding friction F_n' as expressed by Eq. (4). The term coefficient of sliding friction F_s will be reserved for the constant sliding friction resistance which is assumed to be a value of 1.5.

Notation:

F = coefficient of friction between slab and subgrade
F_n' = coefficient of nonsliding friction at any point along slab length
F_n = average coefficient of nonsliding friction

F_s = coefficient of sliding friction assumed to be a value of 1.5
F_a = average coefficient of friction for the case of sliding friction over part of the slab, and nonsliding friction over the remainder of the slab
a' = contraction, in., at any point due to temperature change
x = distance, in., from center of slab, which is assumed motionless, to any point in slab
x_w = distance, in., from center of slab, which is assumed motionless, to point where sliding begins
t = temperature decrease, deg F, from that for slab at rest
e = coefficient of thermal expansion
U = constant establishing the relation between movement and the coefficient of nonsliding friction
L = slab length, in.

The coefficient of nonsliding friction for small movements, as indicated by experimental observations, can be represented by:

$$F_n' = U\sqrt{a'} \quad \ldots\ldots (2)$$

The movement of any point is:

$$a' = etx \quad \ldots\ldots (3)$$

The coefficient of nonsliding friction at any point then is:

$$F_n' = U\sqrt{etx} \quad \ldots\ldots (4)$$

If sliding friction does not take place under any part of the slab, the average coefficient of friction F_n under each half-slab of length $L/2$ is obtained by integration and division by the distance to the center, expressed as:

$$F_n = \frac{2U}{3}\sqrt{\frac{etL}{2}} \quad \ldots\ldots (5)$$

The value F_n given in Eq. (5) is the average coefficient of nonsliding friction for the half-slab length, and this value for F_n may be used as a more refined value in place of F in Eq. (1) where nonsliding friction occurs.

The distance from the center of the slab x_w, at which sliding friction is reached, is obtained from the relationship:

$$x_w = \frac{1}{et} \cdot \frac{F_s^2}{U^2} \quad \ldots\ldots (6)$$

For slabs longer than $2x_w$, sliding friction is assumed to exist near the ends, and nonsliding friction near the center. The average coefficient of friction, as proposed by Kelley, can then be expressed as:

$$F_a = F_s \frac{(1 - 2x_w)}{3L} \quad \ldots\ldots (7)$$

The value F_a given in Eq. (7) is the average coefficient of friction for the case of sliding friction over part of the slab and nonsliding friction over the remainder of the slab. This value may be used in place of F in Eq. (1), where sliding and nonsliding friction occur, as a more refined value for F.

The distance from center slab, x_w, at which sliding friction commences is dependent upon the subgrade constant U, temperature drop, and thermal coefficient, as well as upon the value of the sliding-friction coefficient.

Temperature considerations

Conventional frictional behavior involves reversal of resistance for each reversal of movement. However, with frictional resistance dependent upon amount of movement as well as on its direction, that assumption is not exactly true. Forced movements of short and long slabs show a return movement of 0.01 to 0.02 in. on removal of the force.[5] It can be assumed, therefore, that any slab movement 0.02 in. or larger is actively opposed by frictional resistance. Slab end movements that large would occur in 50 and 100 ft slabs for 5 to 10 F temperature change, which is greatly exceeded by normal daily temperature variations in slabs of reasonable thickness.

Seasonal temperature change is attended by many daily reversals, each subject to frictional resistance. Maximum sustained temperature drop then should result in maximum frictional resistance. This is corroborated by the finding on one continuously reinforced project (Appendix A, p. 354) that seasonal length change was virtually unhindered even in slabs up to 1000 ft long, although extensively cracked. The daily temperature cycle, accordingly, is taken as the basis for determination of frictional stresses.

Daily concrete pavement temperature cycles are related to climatic conditions and slab depth. Daytime maximum slab temperatures seem to vary uniformly from south to north, but nighttime slab temperatures show a somewhat greater drop in the northern regions. Daytime temperaure gradients through the slab depth are on the order of 3 F per in. depth, whereas night temperature gradients are nearer 1 F per in. depth. The slab temperature at middepth is assumed to govern over-all slab movement. The temperature at middepth varies between daytime and nighttime to a greater degree for thin slabs than for thick slabs. From the above established temperature gradients, it is indicated that the maximum daily slab temperature range for design purposes may be reduced 2 F for 1-in. increase in slab thickness.

In the absence of specific information, the following maximum range of daily slab temperatures is suggested for determination of movements:

For southern U.S. and coastal areas	4-in. slabs	35 deg
	8-in. slabs	30 deg
	12-in. slabs	25 deg
For northern U.S. and interior	4-in. slabs	40 deg
	8-in. slabs	35 deg
	12-in. slabs	30 deg

Determination of *U* values

In Appendix A, values of the constant U are derived for recorded test data wherever possible. Computed U values range from about 2, for reversing movements of large slabs on sand cushion, to 10 or more for tests on small concrete specimens 6 to 10 in. thick placed and moved over prepared subgrades.

The effect of variation in slab depth on the magnitude of subgrade frictional resistance has been investigated in Bureau of Public Roads tests on small slabs

6 to 8 in. thick on silt loam soil.[5] The data obtained (see first reference in Appendix A) together with those from two tests of the New Jersey State Highway Department on 9- and 12-in. slabs (also see Appendix A) lead to the tentative conclusion that subgrade modulus of frictional resistance for pavements of different depths varies approximately as the inverse ratio of the square roots of pavement depths.

Following are tentatively proposed values of U in predicting nonsliding coefficient of friction for different subgrades and pavement thicknesses (see Appendix A):

Subgrade condition	U values for given pavement depth		
	4 in.	8 in.	12 in.
Highly cohesive and dry subgrades, and stiff compacted rough gravel or stone subgrades	7	5	4
Average silt, loam, and clay subgrades; normally wet	6	4	3.5
Sand subgrades and cushions, and subgrade papers on smooth bases or subgrades	4	3	2.5

For parts of pavements subject to sliding, a friction coefficient of 1.5 is tentatively proposed, applicable to parts moving in excess of 0.09 in. for $U = 5.0$, to 0.25 in. for $U = 3.0$, for 8-in. highway slabs.

For a night temperature drop of 30 F and a thermal coefficient of 0.000005 (typical values), the distance x_w and the corresponding slab length as well as the slab movement at which sliding at a friction coefficient of 1.5 would commence for some probable values of U are shown below:

Values of U	3	4	5	6	8
x_w, in.	1670	940	600	415	235
L, ft	280	156	100	69	39
a', in.	0.25	0.14	0.09	0.06	0.035

Movements up to ¼ in., without free sliding, have been recorded in friction tests on long slabs. For short slabs, free sliding has generally taken place at movements from 0.05 to 0.10 in. The data seem to indicate that sliding friction probably does not occur under slabs of normal highway dimensions for daily temperature cycles.

For smaller than sliding movements, determination of frictional tension stresses in the slab and its reinforcing at a crack near the slab center can readily be made by substituting values for F_n' from Eq. (2) for F in Eq. (1).

Fig. 1 shows concrete tensile stress and slab-end movements in 20 to 200 ft long slabs for 30 and 40 F temperature drop and 0.000005 thermal coefficient, for U values applicable to highway pavements, and sliding friction coefficient of 1.5.

In cases where nonsliding friction occurs, or where slab length L is equal to or less than $2x_w$, the maximum tensile stress in the concrete which occurs at

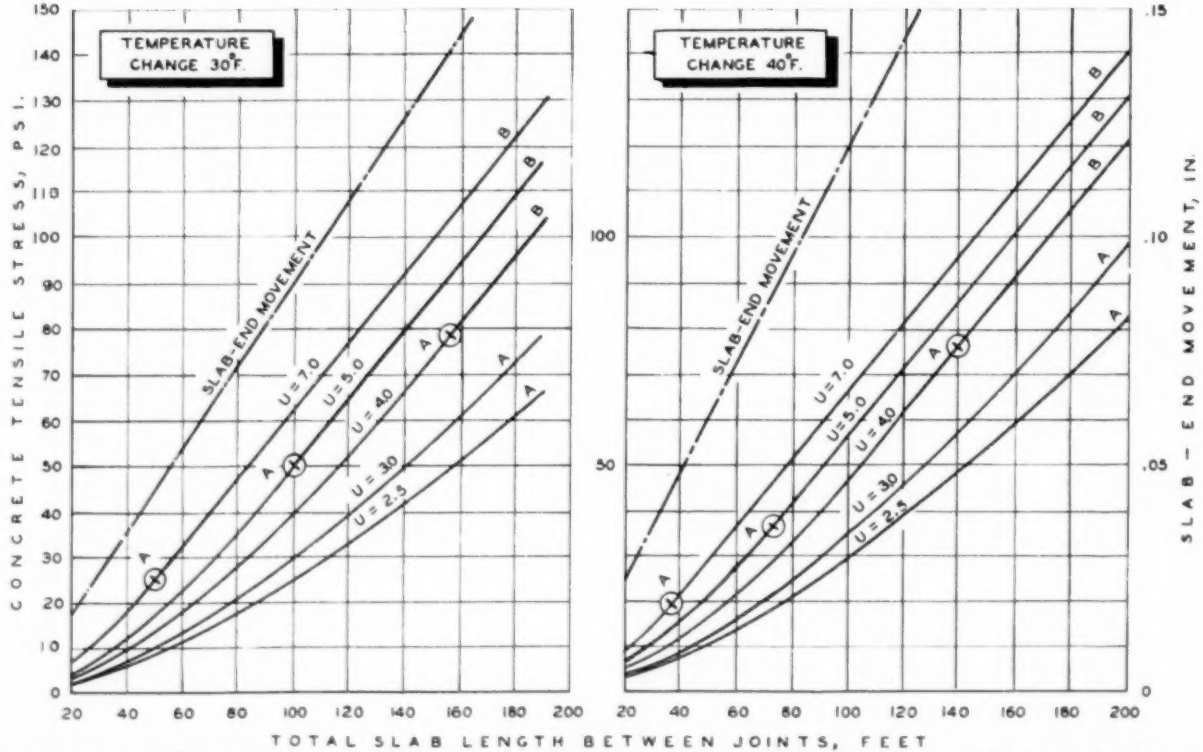


Fig. 1—Frictional tensile stress in concrete and end movement in uncracked pavement slabs up to 200 ft long for 30 and 40 F temperature drop, unadjusted for elastic strain. Based on a coefficient of nonsliding friction in accordance with Eq. (4), and a constant coefficient of sliding friction of 1.5. Concrete weight is assumed at 144 lb per cu ft. Curve 0 to ***A*** for case of nonsliding friction, when $L \leq 2x_w$, Eq. (8). Curve ***A*** to ***B*** for case where sliding exists over part of slab, when $L > 2x_w$, Eq. (9)

the center of the slab may be computed by using Eq. (8), which has been obtained from Eq. (5) and (1).

$$f_c = \frac{UL}{36}\sqrt{\frac{etL}{2}} \quad \text{.....(8)}$$

For the case where sliding friction occurs over part of slab, or where L is greater than $2x_w$, the maximum tensile stress in the concrete which occurs at the center of the slab may be computed by using Eq. (9), which has been obtained from Eq. (7) and (1).

$$f_c = \frac{F_s L}{24}\left(\frac{1 - 2x_w}{3L}\right) \quad \text{.....(9)}$$

REINFORCEMENT STRESSES

Crack opening and steel stress relations

Cracks open in some degree because of stretch of the reinforcement across the crack, directly related to the stress in the steel and the manner of transfer of stress from the steel to the concrete by bond or by welded cross-wire anchorage on each side of the crack. To preserve aggregate interlock across cracks, as well as for effective sealing against water and soil infiltration through cracks, this widening of cracks at low temperature must be small.[4]

Permissible limits for crack openings have not been determined. Assuming normal slopes of 45 deg at interlocking projections of irregular aggregate surfaces, the maximum openings should be something less than the maximum desirable vertical pavement wheel-load deflection on the subgrade, so that aggregate interlock may become effective at less than maximum pavement deflection. Based on maximum pavement load deflections of 0.02 in. (Fig. 110 of reference 6) horizontal crack widening of 0.01 in. due to reinforcement tension, would seem to be a permissible limit.

In Appendix B, available research material on bar and wire bond and stress relations has been analyzed. Available test information is not exhaustive, and does not include reversed and repeated stresses pertinent to contracting and expanding slabs. Insufficient information is available to permit differentiation between wire fabric and other reinforcing material. For weathered steel surfaces, wire stress at a crack appears to be transferred to the concrete predominantly by bond. Welded cross wires may be of considerable value in limiting crack openings if bright wire fabric is used, and for the repeated stress conditions.

Design stresses in crack-control reinforcement

Reinforcing steel for concrete pavements is generally cold-drawn wire or hot-rolled plain or deformed bars of intermediate or hard grade, or rail steel. Structural grade is at times used for tie bars. Sizes under ⅜ in. are not available deformed.

Permanent elongation of the steel is undesirable. Although structural safety is not involved, the steel stress should be limited to prevent yielding at ex-

TABLE 1—REINFORCEMENT DESIGN STRESS

Type of reinforcement	Cold-drawn wire	Hot-rolled bars		
		Structural grade	Intermediate grade	Hard grade*
Yield stress, psi	56,000	33,000	40,000	50,000
Proposed design stress, psi, for given diameter				
under ⅜ in.	35,000	20,000	25,000	35,000
⅜ in.	35,000	20,000	25,000	35,000
½ in. plain	25,000	20,000	25,000	25,000
½ in. deformed	—	20,000	25,000	35,000
⅝ in. plain	—	20,000	20,000	20,000
⅝ in. deformed	—	20,000	25,000	30,000

*Stresses for hard-grade hot-rolled bars apply also to rail steel with 50,000 psi minimum yield point stress.

ceptional continuous temperature decrease, or for subgrade friction higher than assumed in design. A safety factor of 1.5 for yield strength steel stresses is proposed.

Tentatively proposed design stresses for various grades and different sizes of bars or wire used as crack-control reinforcement are given in Table 1. The values are based on 1.5 safety factor for yield-point stress or stress for 0.01-in. crack opening, whichever is the smaller, taken to nearest 5000 psi value.

GENERAL STEEL AND REINFORCEMENT REQUIREMENTS

Reinforcing steel should be furnished in accordance with applicable specifications of the American Society for Testing Materials.

Longitudinal reinforcement

Longitudinal wires or bars are spaced generally not less than 6 in., with the edge members from 2 to 4 in. from the pavement edge and longitudinal joint. Maximum spacing is limited by local stresses, strains, and deformations at each bar with large bar spacings. Tension stresses are imposed on the concrete by the reinforcement with a tendency to secondary cracking perpendicular to highly stressed individual members having high effective bond stress near cracks. These local stresses have not been investigated, but they may be substantial and are additive to load stresses. Maximum spacing should not exceed 15 in.

Spacing of welded cross wires, while perhaps not a cause for concern with weathered wire surfaces, may influence crack opening if bright wire with low bond capacity is used, the crack opening increasing with increased cross-wire spacing. For such conditions, spacing of 12 in. or less may be desirable and apparently should not be exceeded for bright wire fabric.

Continuously reinforced pavements in service with reinforcing bars up to 1-in. size on 6-in. spacing, and with transverse reinforcement of only 1/15 as much weight, have no noticeable longitudinal cracks after 15 years.[2,10] However, large bars exhibit large bond slip and reinforcing stress must accordingly be severely limited when they are used.

The smallest wire generally used for concrete pavements is about 0.20 in. in diameter. Some cases of fracturing of light longitudinal wires at transverse warping joints with the steel continuous through the joint in pavements 8 to 10 years old have been reported.[2,7] A desirable minimum size reinforcing member is clearly indicated.* It is tentatively proposed that longitudinal reinforcing members in highway pavements be not less than No. 3 gage or ¼ in. diameter.

Transverse reinforcement

The foregoing analysis has been specifically related to longitudinal reinforcement; only welded cross-wire anchorage was discussed. In the case of two-lane pavements, transverse warping stresses due to temperature variation in the slab are relieved by the longitudinal joint, and therefore longitudinal cracking is most unlikely. The cross-sectional area of transverse steel based on design consideration is generally exceeded in practice because of other considerations such as construction handling and convenience. The spacing of transverse wires may be controlled by design consideration of cross-wire anchorage for longitudinal steel.

In multiple-lane pavements, with the individual lanes tied by closely spaced tie bars, substantial transverse stresses due to contraction and expansion may occur. The tie steel between lanes is dimensioned to withstand transverse frictional resistance of all lanes to the nearest free longitudinal joint, or edge. The transverse steel reinforcement in each lane should not be less than the amount of tie steel provided across adjacent longitudinal joints. Transverse restraint to free warping at heavily-tied longitudinal joints in such multiple width pavements may increase the tendency toward unforeseen longitudinal cracking.

Heavy-edge reinforcement

Wire-fabric reinforcement for highway pavement slabs has at times been specified with one to four edge wires heavier than the other longitudinal wires. The edge wires are intended to act as dowels for improved load transfer near the corners of the fractured slabs. The load transfer possible by such dowel action for a modulus of support of concrete, $G = 2{,}000{,}000$, is from 500 lb for 0.25-in. wire to 1300 lb for 0.40-in. wire, for 0.01 in. deflection across the crack. The corresponding flexural stresses are about 50,000 and 40,000 psi in the steel, and the concrete bearing stress is 10,000 psi.[8] Several such heavy edge wires may provide some load transfer at differential deflection across a crack smaller than that for which effective aggregate interlock may be established.

Placement of crack-control reinforcement

Cracks, like joints, are subject to temperature warping, causing relatively closer fit across the top of the crack during daytime and across the bottom at

*Fracturing of small reinforcing members is believed due to stresses incident to differential vertical deflection across a crack in establishing aggregate interlock. In addition to tension, the steel is stressed in bending across a crack. The steel flexural stress is proportionate to the deflection and increases rapidly for reinforcing members below ¼ in. diameter. For member size under 0.2 in., bending stress equal to or higher than yield strength can apparently be caused by 0.01-in. deflection across a crack.

night. To preserve maximum possible aggregate interlock at night when cracks are widest, crack-control reinforcement should be placed above center depth of the slab. This follows normal construction practice employing the strike-off method of placement at about 1/3 the pavement thickness from the top.

DISTRIBUTED STEEL PERFORMANCE

Existing highway practice

Common reinforced pavement designs have included 20 to 100 ft long slabs, generally with 0.4 to 0.6 lb per sq ft of steel for 7 to 9-in. pavement slabs up to 50 ft long, and 0.7 to 0.9 lb per sq ft steel for thicker 100-ft slabs. Normally about 2/3 of the steel has been longitudinal reinforcement. Corresponding reinforcement areas are from 0.1 percent for short and medium length slabs, to 0.15 percent for the longest slabs.

Some conventional designs appear to be based on sliding friction coefficient and steel stress approaching yield strength. Existing pavement reinforcement has apparently given satisfactory service, except where the lightest fabric has been used in short slabs. Actual friction is probably lower than assumed, so that, fortunately, actual steel stress is probably also lower. Design considerations approaching true conditions would permit better appraisal of the influence of slab length.

Cracks in reinforced pavements

Pavement crack surveys show fairly few cracks in relatively short reinforced slabs. On over 200 miles of pavement in Louisiana with joints 30 to 50 ft apart, less than 10 percent of the reinforced slabs were cracked.[3] On 15 miles of pumping Illinois pavements, 59 percent of 50-ft reinforced slabs averaging 7 years old were cracked. It is indicated that distributed reinforcement for crack control is increasingly important as slab lengths exceed 50 ft. On the basis of minimum total number of cracks and joints, slab lengths should be greater than 50 ft for reinforced pavements.

In continuously-reinforced pavements,[2] slab lengths adjacent to the ends average from 50 to 150 ft. Design considerations applicable to distributed reinforcement for crack control appear to apply for slabs between 100 and 200 ft long.

Design comparison

Applying Eq. (7) and (1) for U equal to 5, and for 30 F daily temperature cycle and 35,000 psi steel stress, gives 0.05 percent longitudinal steel in a 50 ft long slab and 0.14 percent steel in a 100-ft slab if joint dowels were free; but 0.12 and 0.22 percent, respectively, when joint-dowel sliding resistance is equivalent to 25 psi concrete stress. Without considering dowel restraint, the proposed design considerations indicate less steel than now often used but, as illustrated, the steel requirement in short slabs may be more than doubled by transverse joint dowels which do not slide freely.

Value of crack-control reinforcement

Reinforcement is effective in preventing pumping at cracks. Cracks due to pumping occur 5 to 10 ft from transverse joints. Long-slab reinforcement, dimensioned for stress conditions at much greater distance from joints, provides more steel than needed for crack control near joints.

Pavement blow-ups are significantly decreased by reinforcement.[3] Increasingly, concrete pavements are built without expansion joints.[9] Any crack in a pavement, regardless of orientation, is by reinforcement arrested as a center of potential pavement deterioration; the number of cracks may be significantly fewer. These intangible advantages may add useful pavement life and major economies on heavy-service concrete pavements.

CLOSURE

With respect to subgrade frictional resistance, further information is needed on all variables, effect of subgrade composition and surface, effect of slab movement magnitude, and effect of slab thickness. The influence of frozen subgrade, adhering to the slab firmly according to small scale tests, can now only be suggested by field data indicating that winter freezing is not a critical period for crack development.

The need for limiting allowable steel stress to keep cracks below some acceptable maximum width is indicated by conventional bond tests. The performance of the steel in bond near cracks in pavement slabs, subjected to repeated and progressive influences, may be very different from indications of single pull-out bond tests. Crack infiltration and vertical deflections between crack faces and abutting slab ends both have effects on the steel, presently unknown but important in design. What constitutes an acceptable crack width, in this report assumed to be 0.01 in., is an unexplored question.

Adequate designs have apparently resulted when a value of subgrade friction has been assumed which appears to be higher than the friction indicated for the range of movements normally occurring in slabs of conventional dimensions. Fortunately, the choice of high friction coefficients has provided a margin of safety which allows for the unknown restraints of abnormal slab movements and joints, or a counterbalance for the normally chosen steel design stress. Such empirical designs can be justified only in the absence of more comprehensive information concerning the variables discussed in this report.

APPENDIX A—DATA ON SUBGRADE RESISTANCE TO PAVEMENT CONTRACTION AND EXPANSION

This study is an analysis of information covering the coefficient of friction resistance of the subgrade to movements of concrete pavements due to temperature contraction and expansion. Subgrade frictional tests have been made on small and large slabs, recorded in the literature, or made available for study.

Indirect information is available from observations on concrete pavements in the field. Field observations are limited to some specific types of subgrades and do not include direct measurement of frictional stress in conventional slabs. Data and conclusions from available tests and observations have been summarized in this appendix.

Friction tests

This information covers experiments of moving concrete slabs of different sizes across subgrades of different types, for varying speeds and magnitude of movement. In no case has the movement of a pavement slab, contracting or expanding from its center, been fully duplicated.

Bureau of Public Roads tests—L. W. Teller and Earl C. Sutherland[5] report movement of small 4 x 4-ft slabs, from 2 to 8 in. thick, placed on a natural silt loam, A-4 subgrade. Force and displacement were observed for slowly applied force, and movement up to 0.1 in. The principal findings were:

a. Frictional resistance increased with increasing movement, to sliding at a movement of about 0.05 in., and a constant frictional coefficient thereafter. Before sliding, the resistance increased at a parabolic rate, proportionate to $\sqrt{a'}$, where a' is the movement in in.

b. The subgrade participated in the movement to some extent, but soil movements were reduced by the resistance of subgrade surrounding the small slabs on all sides, as shown by one test made with a groove 5 in. deep cut vertically into the subgrade just ahead of the leading edge of the slab. (Resistance of surrounding soil would be negligible for contracting pavement slabs.)

c. Friction decreased with repeated movements, to one-half the friction value for the first movement. The nonsliding friction for repeated movement of a 6-in. slab approximated $7 \times \sqrt{a'}$.

d. The coefficient of friction increased with decreasing slab thickness for tests on 8-, 6-, 4-, and 2-in. thickness. For 4- to 8-in. slab depths the coefficient of resistance varied approximately as the inverse ratio of the first root of the depths.

Based on these tests, a sliding friction coefficient of 1.5 has been proposed for highway pavement design.

State of New Jersey tests—(Unpublished data made available to ACI Committee 325 by William Van Breemen of the New Jersey Highway Department.) These tests included forced movement of 15 in. wide, 6 ft long slabs, 9 in. thick (also one 6-in. and one 13-in. slab), placed on prepared subgrades within concrete retaining walls 3 to 12 in. away from the specimens. The tests included many types of subgrades and subgrade treatments. The principal findings were:

a. Free sliding normally occurred at from 0.03-in. to 0.06-in. movement, with a friction coefficient between 1 and 2, without subgrade paper. Nonsliding frictional resistance generally increased at parabolic rate, from $6\sqrt{a'}$ for sand and dry loose gravels, to $13\sqrt{a'}$ for clays and damp gravels.

b. High frictional coefficients were observed on bank run gravel, slightly damp, and on damp silty clay, from 2.5 to 3.0 for sliding.

c. Frictional coefficients with subgrade paper were substantially less than without subgrade paper, with the least improvement on concrete sand subgrade, and the greatest improvement on bank run gravel and on

TABLE 2—PRINCIPAL FRICTION DATA FROM MISSOURI FRICTION TESTS*

Place and test	Slab length moved, ft	First movement			Second movement†			Third movement		
		Pre-sliding		Sliding	Pre-sliding		Sliding	Pre-sliding		Sliding
		Move, in.	F_s	F_s	Move, in.	F_s	F_s	Move, in.	F_s	F_s
Martinsburg	100	0.1	$5\sqrt{a'}$	1.5	> 0.5	$2.2\sqrt{a'}$	—	> 0.3	$2.0\sqrt{a'}$	
Eldorado Springs										
A	60	< 0.04	$8\sqrt{a'}$	1.7						
B	60	< 0.10	$10\sqrt{a'}$	> 3.0						
C	60	0.07	$7.5\sqrt{a'}$	2.1						
D	60	—	—	—	0.04	$3.5\sqrt{a'}$	1.0			
F	30	0.05	$20\sqrt{a'}$	5.0						
G	30	—	—	—	9.25	$9\sqrt{a'}$	4			
H	30	0.02	—	5	—	—	—	0.3	$5\sqrt{a'}$	< 3
K	30	0.10	—	5	0.18	$3.5\sqrt{a'}$	—			

*Notation: F_s = coefficient of sliding friction; F_s = coefficient of nonsliding friction; a' = movement of slab.
†A reverse movement toward or beyond original slab position.

damp silty clay. The sliding friction coefficient with subgrade paper was about 1.0, generally at less than 0.02-in. movement.

d. Second-movement friction was less than the first, except for damp silty clay.

These tests on small slabs agree substantially with Bureau of Public Roads tests. They show the value of sandy subgrades and of subgrade covers in decreasing subgrade resistance. Resistances for 13- and 9-in. slabs are approximately in ratio of $\sqrt{9} : \sqrt{13}$.

Missouri tests—These tests were performed in conjunction with stress-curing experiments and construction (see *Engineering News-Record*, Nov. 9, 1939, p. 68), on projects at Martinsburg and Eldorado Springs, Mo., under supervision of the Missouri State Highway Department. The tests have been compiled and made available to ACI Committee 325 by Bengt F. Friberg.

Highway pavement slabs from 30 to 100 ft long, with open joints and "Translode" doweling, were moved back and forth on the subgrade by means of pneumatic pressure applied in cells in the joints. The pressures generally were imposed rapidly; movements ranged up to about ¼ in. The tests were made before complete curing of the pavement.

The pavement at Martinsburg was placed on asphalt-impregnated subgrade paper, that at Eldorado Springs generally on clay subgrade. A summary of the Missouri tests is given in Table 2.

The principal findings of the Missouri tests are:

a. The coefficient of sliding friction was between 1.5 and 2 for initial movement of 60- and 100-ft slabs, but higher for 30-ft slabs. The resistance to movement was progressively smaller for each reversal of movement, and sliding did not then take place until large movements had occurred.

b. For repeated nonsliding movements, the coefficient of nonsliding friction increased at a linear or parabolic rate with movement, in accordance with the relationship $2\sqrt{a}$, for slab on subgrade paper, and $3.5\sqrt{a}$, minimum for slabs on clay subgrade.

c. By the imposition of pressures in two adjacent joints, large reversal of movements of the intermediate slab could be accomplished with small differential in pressure, at a coefficient of friction less than 1 for 60-ft slabs, and less than 2 for ⅛- to ¼-in. movement of a 30-ft slab.

The Missouri tests indicate that frictional resistances under a slab of normal highway dimensions are lower than found in tests on small slabs, especially for cyclic movements of expansion and contraction of pavement slabs.

Kansas tests—"Lawrence Experimental Concrete Pavement, A Twelve-Year Study," by the State Highway Commission of Kansas (1949), reports a few frictional tests, covering resistance of six 50-ft highway pavement slabs 7 in. thick. To induce movement, the force was applied by jacking adjacent slabs in an open intermediate joint. Two slabs were placed on a ½-in. sand gravel cushion, the others on existing subgrade, of both normal and excessively wet A-6 to A-7 type, with subgrade paper between the slab and the subgrade in all cases. The tests were made at 7- to 13-month age, with two slabs retested 1 year later. Only one test was made on each slab to establish resistance.

Slab movements of 0.06 in. to over 0.30 in. were obtained, with free sliding at a coefficient of friction of 1.3 on normal subgrade, 1.4 on ½-in. sand cushion, and 1.8 to 2.0 on excessively wet subgrade at movements approximating 1/10 in. Retests 1 year later gave the same or slightly lower resistance values. Rebound movement during removal of jack pressure was from 0.013 in. to about 0.03 in.

Nonsliding frictional resistance increased at more rapid rate with movement than

obtained with Eq. (4). The resistance for 0.035-in. movement (approximate maximum movement for daily temperature cycle) is about $7\sqrt{a'}$. No short-time repeat tests were made.

Field observation data

Some information of indirect nature on subgrade frictional resistance is available in the literature, from continuously reinforced concrete pavement installations and from pavement prestressing experiments.

Prestressed pavement installation—Data bearing on concrete pavements on sand subgrade have been obtained in construction of prestressed concrete road slabs at Crawley, England, reported by McIntosh and Mercer.* For a 6-in. slab over 400 ft long on a sand cushion, prestress was applied to overcome friction to its center at a coefficient of friction of 0.5. The force was applied as post-tensioning, and closed open cracks which had occurred before prestressing either immediately or after the first night. Cracks remained closed thereafter.

Continuously-reinforced-pavement observations—Accounts of extensive field observations on large highway projects have been published in *Proceedings* of the Highway Research Board (by H. D. Cashell and S. W. Benham,† for an experimental pavement in Stilesville, Ind.; by H. W. Russell and J. D. Lindsay in V. 30, p. 45, for a pavement in Illinois; by T. E. Stanton in V. 30, p. 28 for a pavement in California; and by W. Van Breemen in V. 30, p. 61, for a pavement in New Jersey). The first three projects covered performance with bar-reinforced slabs; the first and last projects include information on fabric-reinforced pavements as well. The published accounts have been studied with particular reference to subgrade resistance.†

Cashell and Benham show that *seasonal* length changes of 2 in. or more are not diminished by frictional resistance under slabs up to 1000 ft long even though closely-spaced transverse cracks would permit strain for low stress in the steel crossing the cracks to equal temperature contraction.

Contraction of slab ends for normal night temperature drop seldom exceeds ¼ in., and is restrained in the 200 to 300 ft of pavement nearest the ends, showing that frictional resistance attends daily contraction and expansion of long highway pavement slabs.

For 20 F night temperature drop and about 0.15-in. end movement, the average coefficient of friction under a 300-ft active slab end is indicated to be about 0.5, which would correspond to the relationship $2.6\sqrt{a'}$ on the silt-loam subgrade at Stilesville.

Conventional slab lengths and movements—In slabs of conventional lengths there is little decrease in temperature contraction because of concrete strain. Contractions and expansions for daily temperature cycles of 30 F vary from 0.02 to 0.10 in. at ends of 20- to 100-ft slabs, respectively. Maximum coefficient of friction values at slab ends F_a' and average coefficient of friction values F_a under each half-slab are presented in Table 3, under consideration of coefficient of friction for free sliding of 1.5.

According to the tests and field observations reported upon above, it seems unlikely that friction under highway slabs would be greater than $7\sqrt{a'}$. The evidence points to the likelihood that it is much less, especially for long slabs, and for normal granular subgrade.

*McIntosh, J. W., and Mercer, Jack, *Proceedings*, Institution of Civil Engineers (London), V. 1, No. 1, Part II, Feb. 1952, p. 227.

†Friberg, B. F., "Frictional Resistance Under Concrete Pavements and Restraint Stresses in Long Reinforced Slabs," *Proceedings*, Highway Research Board, V. 33, 1954, pp. 167-182.

TABLE 3—COEFFICIENTS OF FRICTION FOR VARIOUS SLAB LENGTHS AND VALUES OF *U*

30 F temperature drop and coefficient of expansion 0.000005

	Values of U		
	2	5	10
$L = 30$ ft; $a' = 0.03$ in.			
Pre-sliding			
F_n' at slab end	0.35	0.85	—
F_n (average)	0.23	0.60	—
Sliding			
F_s at slab end	—	—	1.5
F_s (average)	—	—	1.1
$L = 60$ ft; $a' = 0.06$ in.			
Pre-sliding			
F_n' at slab end	0.50	1.2	—
F_n (average)	0.35	0.8	—
Sliding			
F_s at slab end	—	—	1.5
F_s (average)	—	—	1.3
$L = 100$ ft; $a' = 0.10$ in.			
Pre-sliding			
F_n' at slab end	0.65	1.5	—
F_n (average)	0.40	1.0	—
Sliding			
F_s at slab end	—	—	1.5
F_s (average)	—	—	1.4

Summary of published information

Frictional stresses in conventional pavements, in reinforcement crossing cracks, and in tie bars crossing longitudinal joints, are normally smaller than computed for a constant coefficient of friction of 1.5.

Friction between the slab and the subgrade may be considered to develop in accordance with $F_n' = U\sqrt{a'}$; where F_n' is the coefficient of nonsliding friction, a' the movement in in., and U is a constant establishing the relation between movement and the coefficient of nonsliding frictional resistance, dependent upon subgrade, and pavement thickness; possibly also upon pavement length.

Values of U indicated by past exploratory research, and applicable to daily contraction and expansion of large pavements in the field, are listed below:

For thin slabs on stiff subgrades (These may be cohesive or wet gravelly surfaces)	$U = 5$ to 7
For average highway pavements on silt, loam, or clay subgrades	$U = 3$ to 4
For pavements on sand cushions of some thickness on subgrade paper (This value may apply also for thick pavements, as used for airfield loadings)	$U = 2.5$

Sliding friction, with a coefficient of friction of 1.5, would apply for parts of pavements subject to actual sliding.

APPENDIX B—REINFORCEMENT STRESSES AND BOND AT CRACKS IN CONCRETE PAVEMENTS

General characteristics

The function of reinforcing steel provided for crack control in concrete pavements is to keep cracks substantially closed, so that the cracks are sealed against serious moisture and soil infiltration, and so that aggregate interlock across the cracks is maintained without appreciable vertical play at the lowest temperatures.

Steel in tension across a crack stretches elastically for some distance on each side of the crack as the tension is gradually transferred from the steel to the concrete by bond. The stretch is evident in opening and closing of cracks, and should not exceed desirable limits of small magnitude. Permissible maximum

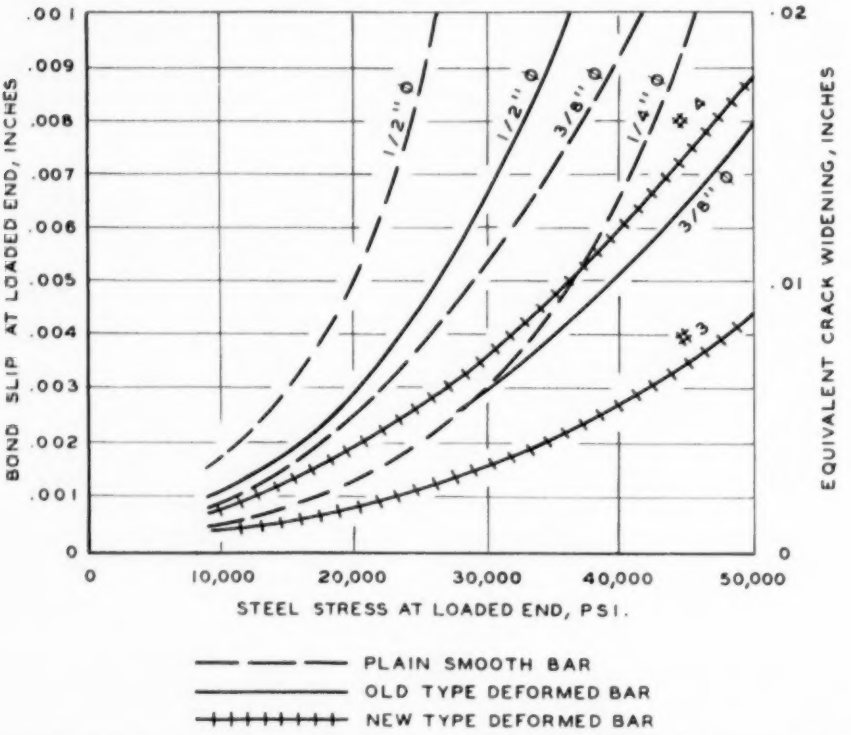


Fig. 2—Relationship between tensile steel stress in pavement reinforcement and crack widening for several sizes of plain and deformed bars, as suggested by observed slips at loaded end of conventional bond tests

tension stresses in pavement reinforcement may accordingly be dependent upon bonding characteristics of the reinforcement as governed by member size and surface and by cross-wire anchorage and spacing.

The literature contains few experimental data applicable to pavement reinforcement. Data in this appendix are from readily available sources without claim to inclusiveness. The information is presented to illustrate the conditions for consideration, rather than as a specific guide to design. The few tests show a degree of uniformity, but it must be remembered that applicable data for repeated and reversed stress conditions are altogether lacking.

Bond and slip characteristics

Curves of longitudinal deflection of bonded bars in Fig. 2 have been derived from conventional bond tests of Gilkey and Chamberlin, Watstein, Clark, Menzel, and others, which show slip at loaded end of reinforcing pull-out tests.

Experimental data are assumed to be applicable to pavement concrete, without longitudinal stress in the concrete at the face of the crack. Crack widening is defined as longitudinal deflection of the pavement reinforcement across a crack, and is assumed to be twice the slip at the loaded end of bond tests. Crack openings up to 0.01 in. are of primary interest, corresponding to loaded-end slips of 0.005 in.; these are relatively small compared to large ultimate bond-slip observations. The stretch of steel across pavement cracks is therefore assumed to be elastic deflection, as seems possible also for bond slip at these small deflections.

The increasing rate of slip with increasing load in bond tests, suggests increasing length as well as stress in the steel stretching to produce the slip. The relationship between observed steel stress and slips corresponds approximately to high and relatively constant bond stress existing over a short distance near the loaded end, the distance increasing in direct proportion to the load.

The anticipated crack widening is assumed to follow the same relationship:

> Steel stress at a crack is transferred by active bond at a high specific bond stress within a short distance on each side of the crack, irrespective of greater bonded length of steel.

For tension stress f_s at a crack in a bonded steel member with diameter d, and modulus of elasticity E_s, the active length of bond c on each side of the crack, at a high specific bond stress, or bond capacity, u, is:

$$c = \frac{f_s}{4\,\frac{u}{d}} \qquad \text{.....(10)}$$

The constant $4\,u/d$ is termed bond modulus B_m, related to bar surface and size, and possibly to concrete quality. It expresses the increase in steel stress for each inch of active bond. The total stretch of steel across a crack, which is the sum of the stretch on both sides, and assumed equal to the crack widening, D, may be computed:

$$D = 2\,\frac{f_s}{E_s} \cdot \frac{c}{2} = \frac{f_s'}{B_m E_s} \qquad \text{.....(11)}$$

Appraising the few experimental data from Fig. 2 we find the performance to correspond to the following bond capacities (bond stress computed to nearest 100

psi, and bond modulus to nearest 500 psi per in.):

Bar type	Diameter, in.	Bond stress, psi	Bond modulus, psi per in.
Clean plain steel	¼	300	5000
	½	200	1500
Rusty or old deformed bar	⅜	500	5500
	½	300	2500
New type deformed bar	#3 size	900	10,000
	#4 size	500	4000

These values give an indication of the approximate performance which may be expected with various reinforcing materials and sizes. Computed crack openings for different reinforcing sizes and bond capacities or bond modulus are shown in Fig. 3. No information is available on change in bond and slip with repetition of loading. The values of Fig. 3 can be used only as preliminary indications, in the absence of related research.

Using the data at hand, the steel stresses shown in Table 4 are indicated for probable crack widths of 0.01 in. and 0.02 in. Bond stresses are in any case limited by steel yield strength, for which an appropriate safety margin should be applied. Table 4 includes bar sizes larger than generally used for pavement reinforcement as crack control.

Cross-wire anchorage

Wire fabric and welded bar mats are provided with mechanical anchorage of cross wires or bars. This may be helpful in resisting progressive slip under repeated loading, to provide slip resistance for smoooth nonweathered material, and to resist unusual stress peaks.

The only available experimental evidence on influence of cross-wire anchorage on crack width has been assembled in Fig. 4, but the data are obscured by tensile strains and bond of the wire between the crack and the first cross wire on each side, so that the data cannot be interpreted with specific reference to cross-wire anchorage and deflection of the cross wires incident to such anchorage for normal field conditions. The advantage of welded cross wires may be noticeable for repetitive conditions, especially if they are closely spaced.

Cross-wire anchorage is believed to be analogous to loading of dowels, the cross

TABLE 4—STEEL STRESSES IN RELATION TO CRACK WIDTH

Bar type and diameter	Steel stress, psi, for given crack width		Active bond distance, in., on each side of given crack	
	0.01 in. wide	0.02 in. wide	0.01 in. wide	0.02 in. wide
Plain round weathered steel, old style deformed				
¼ in.	50,000 +	—	6	—
⅜ in.	39,000	50,000 +	8	10
½ in.	26,000	36,000	10	15
¾ in.	20,000	28,000	15	21
New style deformed				
⅜ in. (#3)	50,000 –	—	6	—
½ in. (#4)	36,000	50,000	7	10
⅝ in. (#5)	29,000	41,000	10	14
¾ in. (#6)	26,000	36,000	12	17

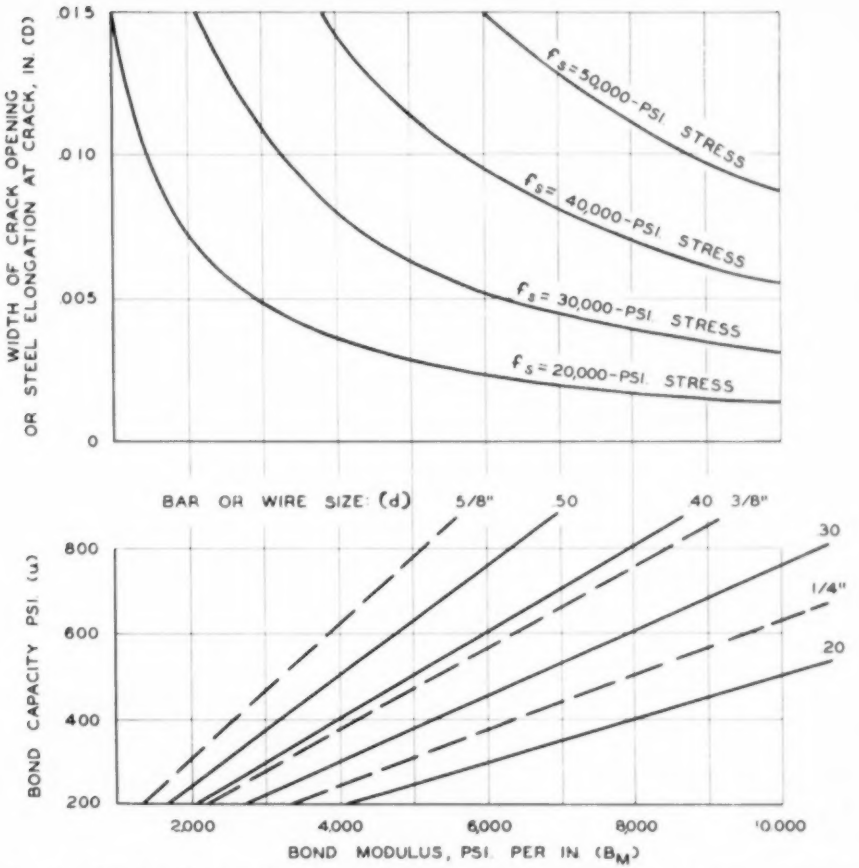


Fig. 3—Crack widening, defined as steel elongation across a crack, corresponding to 20,000 to 50,000 psi steel stress at the crack, for varying bond modulus of the reinforcement. Also shown is equivalent bond performance for different sizes of bars

wire acting as a dowel extending both ways from the weld, and deriving its anchorage from bearing against the concrete on each side of the weld, limited by bending in the cross wire or crushing of the concrete at the weld. Fig. 5 shows computed lengths of cross wire effective as anchorage, anchorage provided at bending stress in the cross wire of 70,000 psi, and the deflection of the cross wire in the concrete to develop this anchorage, for modulus of bearing, G, between cross wire and concrete from 1,000,000 to 4,000,000 psi per in. deflection.

Direct bearing immediately at the weld slightly extends the available cross-wire anchorage. For normal size relations this length may be considered equal to the cross-wire size. Cross-wire anchorage capacity, based on 70,000 psi bending stress (approximate wire yield strength), and deflections for such anchorage, taken from Fig. 5, are given in Table 5 (assuming G = 2,000,000 psi). Fig. 5 is a theoretical derivation for the strength of the cross-wire anchorage based on dowel design formulas.[8]

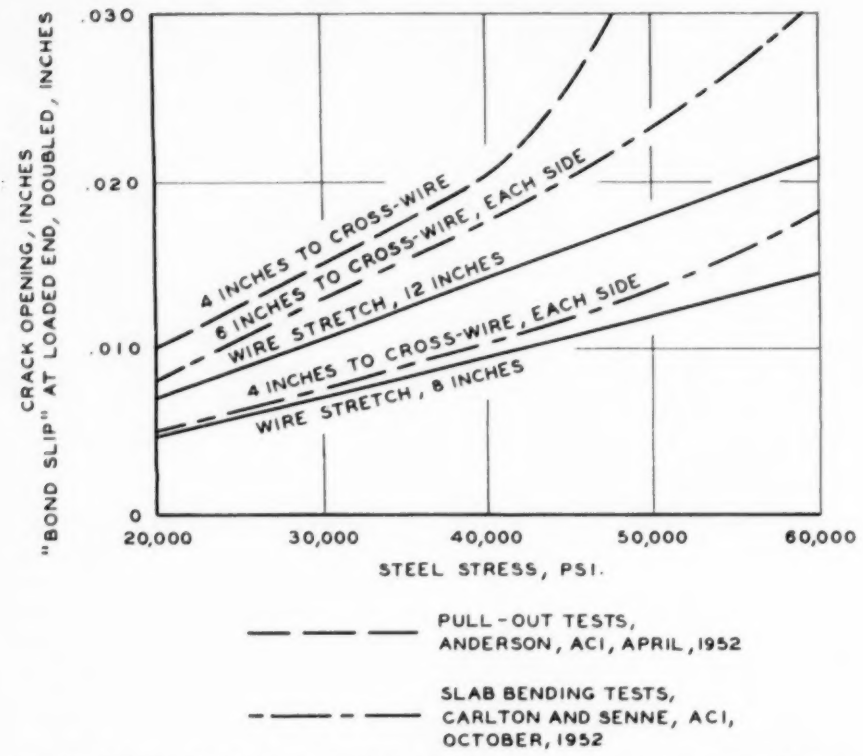


Fig. 4—Relationship between crack widening and steel stress for wire-fabric reinforcement with cross-wire anchorage, derived from published pull-out and flexural tests with wire-fabric reinforcement

No experimental verification of these anchorage values exists. Considerable uncertainty surrounds the value of G, especially for small wire sizes. Yielding may not take place in the cross wire until stresses exceed the tension yield strength by a considerable amount.

TABLE 5—CROSS-WIRE ANCHORAGE CAPACITY*

Cross-wire diameter, in.	Anchorage, lb.	Deflection, in.	Corresponding stress, psi, in longitudinal wire of size		
			0.20 in.	0.30 in.	0.40 in.
0.20	700	0.0020	22,000	10,000	—
0.25	1200	0.0022	28,000	17,000	9,500
0.30	1800	0.0023	58,000	25,000	14,000
0.35	2600	0.0025	—	36,000	21,000
0.40	3600	0.0027	—	—	29,000

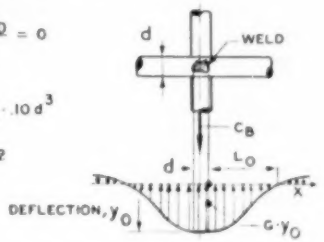
*Based on Fig. 5.

$$y_o = \frac{P_D - \beta M_o}{2\beta^3 E_s I} \qquad \frac{dy}{dx}_{(x=0)} = \frac{2\beta M_o - P_D}{2\beta^2 E_s I} = 0$$

$$M_o = f_y \frac{2I}{d} = f_y .10 d^3; \quad P_D = 2\beta M_o = 2\beta \cdot f_y \cdot .10 d^3$$

CROSS-WIRE ANCHORAGE: $C_B = 2P_D + Gy_o \cdot d^2$

$$\beta = \sqrt[4]{\frac{G\,d}{4E_s I}}$$



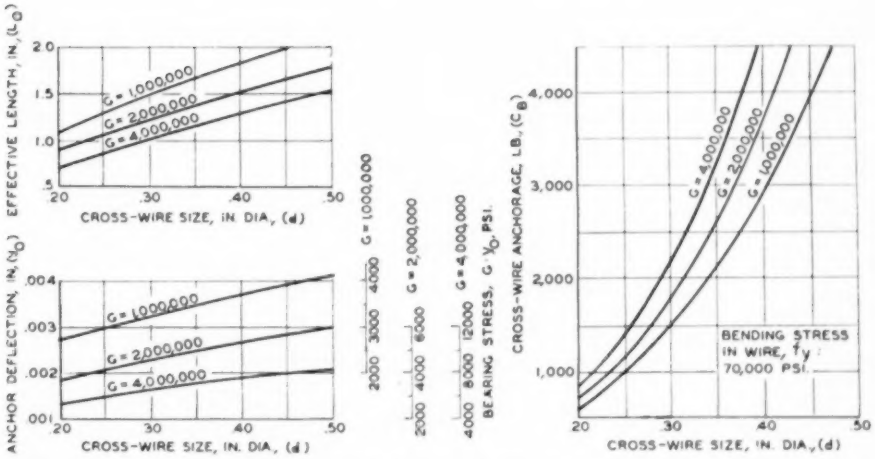


Fig. 5—Theoretical derivation and values of strength of cross-wire anchorage, effective length of cross wire, cross-wire deflection at 70,000 psi, and bending stress in the cross wire, based on dowel design formulas,[a] and for three values of G (modulus of support of the concrete)

An indication of the relative performance of weathered-wire bond and cross-wire anchorage may be obtained from a comparison of the stresses and longitudinal deflections for bonded wire (for 0.01-in. deflection), and for unbonded wire at cross-wire anchorage capacities. This comparison has been made in Table 6 for 0.20-, 0.30-, and 0.40-in. diameter wire sizes, with transverse wires 6 and 12 in. apart.

The comparison would indicate that weathered wire surfaces may transmit by bond on each side of a crack a higher stress in the wires than might be transmitted by cross-wire anchorage alone, and with less deflection. However, the same comparison may not hold true after repeated stressing of the wires.

Experimental research of adequate scope is not available for correlation of this study of stresses and deflections in pavement reinforcement. The data may be considered only as tentative and qualified indications to design of steel reinforcement in concrete pavements.

a. For crack-control reinforcement permissible steel stress is inversely related to bar or wire size.

TABLE 6—COMPARISON OF STRESSES AND ANCHORAGE DEFLECTION FOR DIFFERENT SIZE WIRES

Bonded performance				Anchored performance (no bond)*			
				Cross-wire anchorage		Total deflection for given cross-wire spacing	
Wire diameter, in.	Assumed bond, psi	Stress, psi, for 0.01-in. crack	Bonded length, in.	Stress, psi	Deflection, in.	6 in. on center	12 in. on center
0.20	600	58,000	5.0	22,000	0.0020	0.009	0.013
0.30	500	43,000	6.5	25,000	0.0023	0.010	0.015
0.40	500	33,000	8.0	29,000	0.0027	0.012	0.018

*In case of no bond, the crack is assumed midway between cross-wire spacing of 6 or 12 in. In addition, the longitudinal and transverse wires are assumed the same size. Further, the total deflection includes the deflection of the two anchorages and the elongation of the wire between the anchorages under an equivalent stress.

b. Pending further research, it would seem practicable to use steel stresses holding cracks to .01-in. opening.

c. Permissible reinforcement stresses of 25,000 to 35,000 psi are indicated by available experimental evidence for ½-in. and smaller reinforcement members.

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For such discussion of this report as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-19

Auditorium Framed with Prestressed Roof Girders*

By ERIC C. MOLKE†

SYNOPSIS

Design and construction features of 146-ft roof girders for Springfield, Mo., high school auditorium are discussed. Cast of 5000-psi concrete and prestressed on the ground, the girders were raised between twin concrete columns by the same jacks used for prestressing. Girders were simply supported temporarily while precast roof panels were erected, and frame continuity was later introduced by prestressing the girders to the columns. Stability requirements during construction as well as for the completed structure are considered.

GENERAL CONSIDERATIONS

An auditorium constructed for a high school building in Springfield, Mo., demonstrates that cooperation between architect, engineer, and builder during the early stages of layout can lead to an economical and attractive solution. The architect early accepted the conception that the 146 ft long roof girders would be cast and prestressed on the ground and that they would be raised between twin concrete columns, so that the regular prestressing jacks could also be used for lifting.

The next question was the shape of the girders and columns. From association with monolithic concrete design, the architect expressed a preference for a portal-like frame. He also wanted to use a level roof slab of exposed precast panels at the underside of the roof girder. A compromise was arrived at by using fairly heavy columns, at the same time shaping the beam economically for the lifting condition. The heavy columns invited continuity between columns and roof girders.

Any reasonable amount of continuity moment at the knuckle between girders and columns could be induced by prestressing the connection after the girders were raised. Trial calculations indicated, however, that frame continuity equal to the full live and dead load condition would not only have caused an awkward

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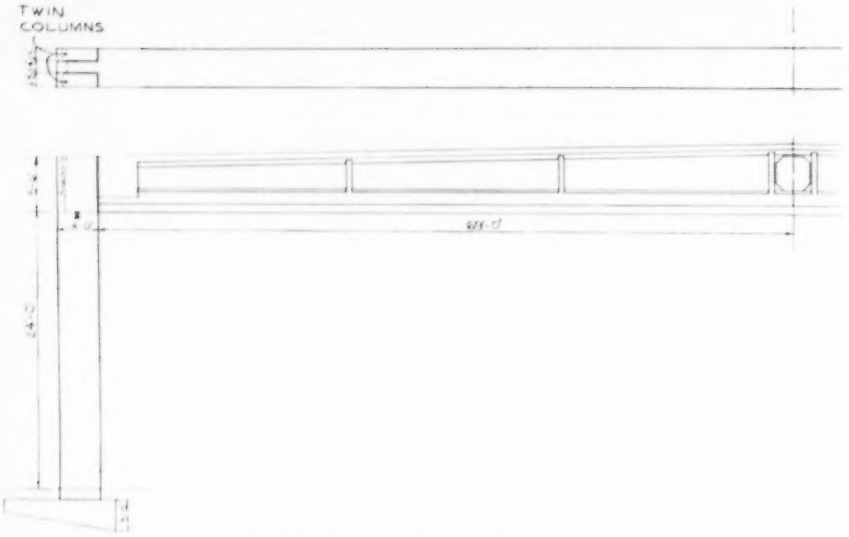

Fig. 1—Top portion is plan view of top flange of girder. Lower part shows section through structure

detail at the knuckle but would also necessitate large columns, footings, and probably tension ties at the footing level. Estimates indicated that the girders would weigh 40 percent and the precast panels 25 percent of the total roof loads.

Estimates also indicated that with total restrainment of the roof girders into the columns for the full live and dead loads, horizontal thrust at footing level would be approximately 100 percent of the maximum column reaction. With restrainment for live load and the dead load from only the roof panels, the thrust would be about 55 percent; with restrainment for live load only the thrust would be about 33 percent of the maximum column reaction.

Foundations were to be spread footings on clay and it was felt that for this condition the ratio between horizontal thrust and vertical reaction should not be more than 1:3. This ratio permitted an economical footing design and it was therefore concluded to induce frame continuity only after the roof panels were erected on temporarily simply supported girders. It was felt that a simple temporary support of the girders could best be achieved by providing a rocker bearing over the slightly rounded top surface of a rail bulb. Fortunately, it was found that the largest AREA rail had just sufficient bending strength to bridge the temporary girder reaction across the space between the twin columns.

Various means of achieving the continuity for live loads between column and roof girder were considered. This could have been done without prestressing by either welding together or lapping bars projecting out of columns and girders. The choice of twin columns to facilitate the lifting did not lend itself too well to such a detail, especially since it was desirable to bring the full column cross

section up to the top level of the roof girders to provide a proper seating surface for the lifting mechanism. A groove and mortise joint as used in carpentry work, with end blocks projecting out of the beam sides and pressed against the column faces, seemed more practical.

DESIGN DETAILS

Fig. 1 shows a typical cross section through the frame structure and a plan view of the top flange of the girder. Fig. 2a is a cross section through the girder near the midspan, and Fig. 2b, an end view at the connection between girder and column.

Columns were conventionally constructed, but of 4000-psi concrete to provide sufficient strength for the final prestressing of columns to girders.

The girders were of 5000-psi concrete. Their top flange was made wide to resist the large positive bending moment at midspan and of equal width at the out-to-out dimension of the twin columns. Note the arrangement of six vertical stiffening ribs per girder and the opening provided in the web at midspan to facilitate walking at the roof level. The bottom flange was shaped to accommodate the placing of the precast roof panels. It was provided with reglets above the slab level to receive the roof flashing.

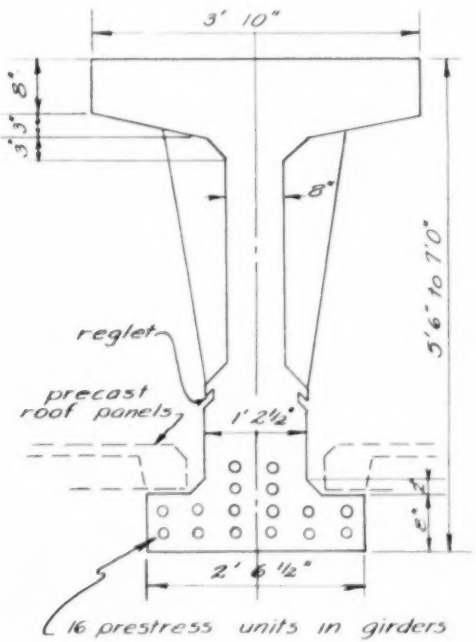

Fig. 2a—Cross section through girder

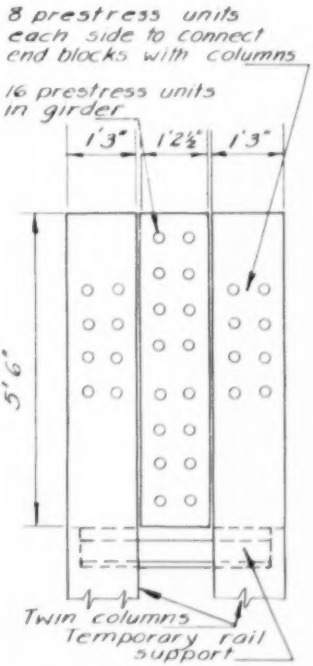

Fig. 2b—End view of girder

Fig. 3—Plywood forms for casting girder. Approximately positioned conduits each contain 12 prestressing wires

Each precast panel had anchor plates cast into the bottom of one rib, to match with similar anchor plates along the bottom flange of the roof girders. These anchor plates were welded together and the joint between end face of panels and girder was grouted to provide a rigid lateral support to the girders.

A web thickness of 8 in. was assumed to keep the principal tensile stresses below the permissible value of 150 psi. This width also permitted "draping" of the 16 prestressing units; each unit consisted of twelve 0.250 in. diameter wires with an ultimate strength of 240,000 psi. Calculations for prestressing were based on design criteria recommended by the Bureau of Public Roads.

Prestressing of the girders to the twin columns was made with similar 12-wire units, 8 ft long, eight units located on each side of the girder. All anchorage hardware was of the so-called "PI" system using two upsets at either end of the wires, one for pulling and one for anchorage, as shown in the illustrations.

Fig. 4—Forms and reinforcement for girder were placed between the already-cast columns

CONSTRUCTION PROCEDURE

Fig. 3 shows one side of the plywood forms in place during girder construction. The bottom soffit had a camber of 2 in. and was placed on 2 x 4-in. sleepers, 6 ft long at 8-in. centers, to distribute the load on the ground floor slab. Flexible conduits of 1⅜ in. inside diameter, each containing 12 prestressing wires, were fastened to stirrups and ties. As shown in this figure, they were first placed approximately. They had to be draped both vertically and horizontally, being located as low as possible in the bottom flange as shown in Fig. 2a and disposed in two vertical rows at the support (Fig. 2b). Fixing cables in exactly scheduled positions was done by tying to stirrups spaced at 18 in. and finally smoothing out curve of cables by tapping units with a mallet.

A ¾-in. space between columns and girders was obtained by lining adjacent column faces with ⅜-in. plywood (Fig. 4) backed by ⅜ x 1-in. steel straps. Plywood in the ¾-in. joint was easily stripped because steel straps were pulled out first. Fig. 4 also shows mild steel reinforcement in the anchor blocks of the girder. Fifty cu yd of concrete per girder was placed with a ½-cu yd bucket on a 20-ton Lorrain crane. It was a 6½-bag mix with a slump of 3 to 4 in. The forms were completely filled at one end and placing progressed to the other end. One electric vibrator was used with a 1¼-in. head for the narrow web, and with a 1¾-in. head for the flanges.

About 7 days after casting when the concrete had obtained a minimum strength of 4000 psi, the girders were completely stressed on the ground. Two 100-ton Simplex hollow core jacks (Fig. 5) with a ½-hp hydraulic pump were used for

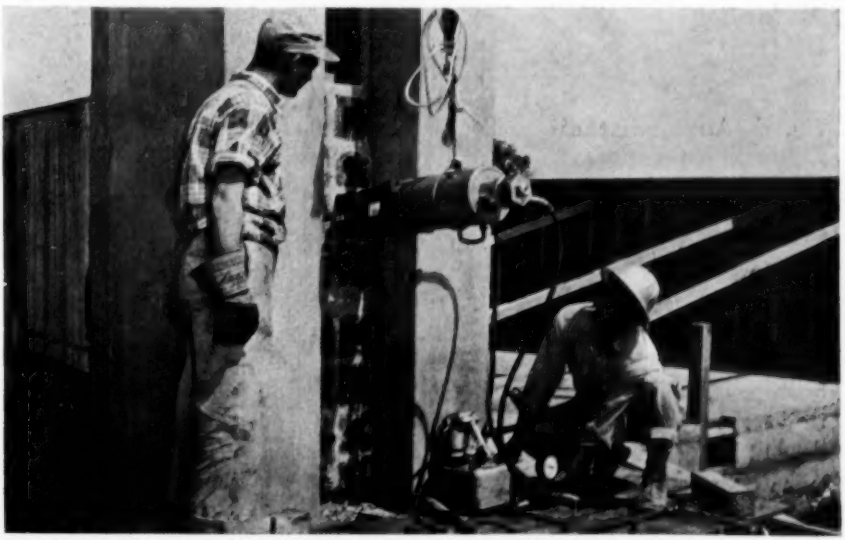

Fig. 5—Girder stripped after casting. Braced forms for another girder are in background. Hollow core jack and hydraulic pump in use for prestressing

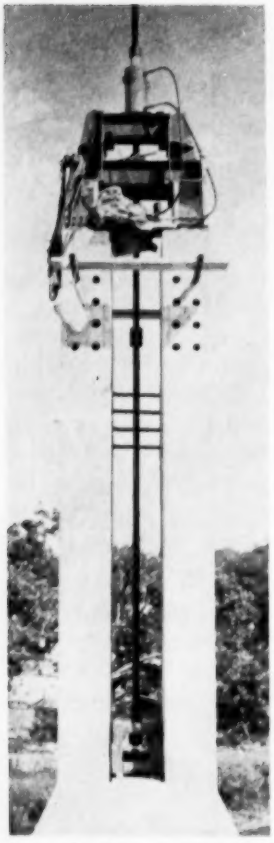

Fig. 6—Arrangement of lifting yoke and rods

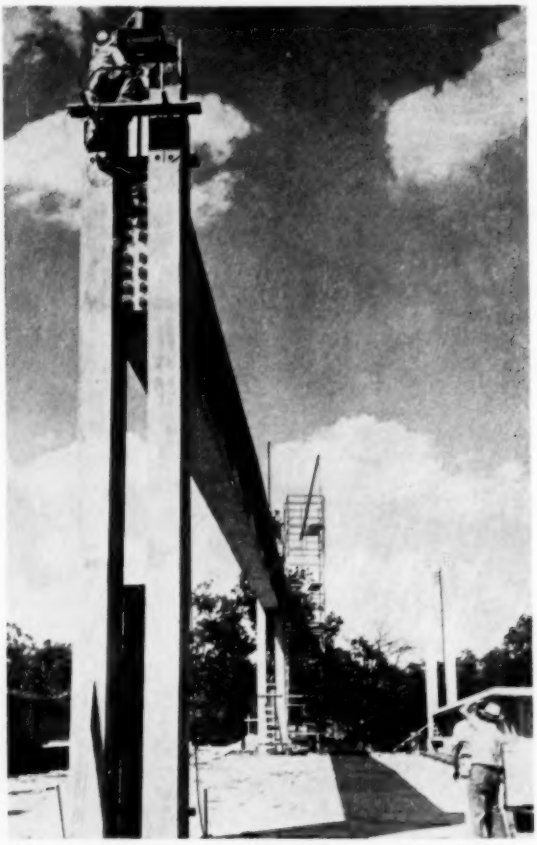

Fig. 7—Girder during lifting. One section of lifting rod and coupler already removed

stressing the wires, one at each end of the units. Total elongation per unit varied between 10½ and 10¾ in., depending on degree of draping. While stressing, the original camber of 2 in. was increased by ⅞ in., but this upward deflection was finally canceled out by the weight of the precast roof panels.

Fig. 6 shows the arrangement of the lifting yoke, the 2-in. lifting rods with ¼-in. Acme thread and a short steel anchor beam at the bottom. The anchor beam served to distribute the pull from the lifting rods into four 1½-in. round removable Richmond anchor bolts which were embedded 11 in. in the concrete. Same jack and hydraulic pump visible on top were used for prestressing (Fig. 5).

Lifting rods were in 20-ft sections, with a coupler as shown. The lifting yoke was made in two tiers; the upper tier was for the support of the jack, while the lower tier served to support a take-up nut on the lifting rod. Note the eight holes at the top in both of the twin columns. Short prestressing units were inserted

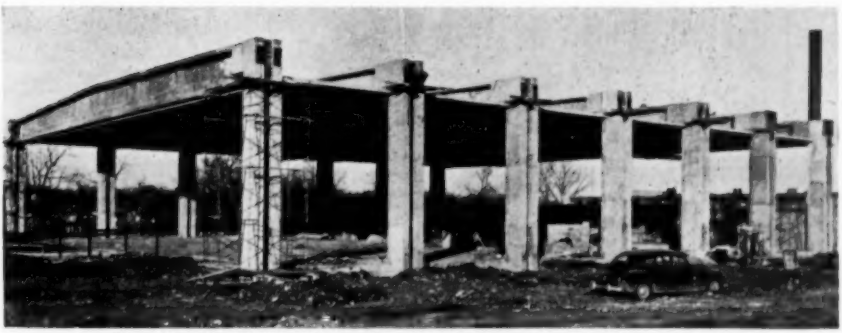

Fig. 8—All girders in position, on temporary simple supports

there later to make connection with the end blocks of the beam.

Fig. 7 shows a girder during lifting. One section of the lifting rod and the coupler which was shown in Fig. 6 have been removed. During removal of the unit the beams were temporarily held on two 8 x 8-in. timber uprights wedged between the twin columns as shown in this figure.

After all girders were raised to proper position (Fig. 8), temporarily supported precast roof panels were placed by crane. A simple plank scaffolding projecting out at either beam end served for placing the end connecting cables. The girders took their free deflection while rocking on the railroad rails inserted between the twin columns.

Girders and columns were finally connected (Fig. 9) by 12-wire units, which were inserted through holes at the top of columns and into corresponding holes in girder end block. The eight short units on each side of the girder were prestressed with a 60-ton jack and hand pump to join girder and column. A hand pump was used since the required elongation was only $\frac{1}{8}$ in.

All units were grouted after prestressing to fill the spaces in the surrounding flexible tubing. Fig. 10 shows the operator standing on the precast roof panels. The space between the end of the roof panels and the girder was tightly filled

Fig. 9—Girders finally connected to columns by stressing end block to columns

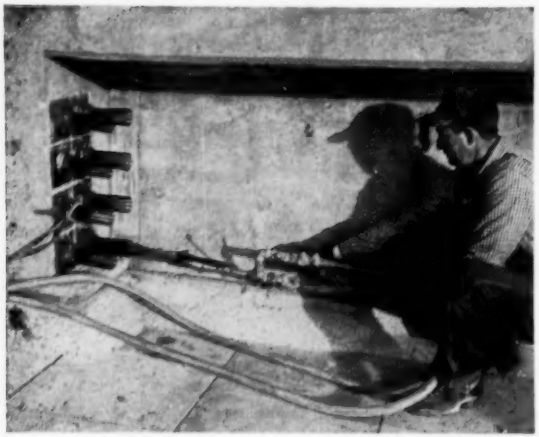

Fig. 10—All units were grouted after prestressing to fill spaces in flexible tubing

with mortar and covered with sloping concrete corner fill which not only provided stability for the girder but also facilitated the application of flashing. The flashing was later caulked into the reglet along the beam, which can better be seen in Fig. 9.

These construction procedures made it possible to raise the 104-ton girders by a simple application of regular prestressing equipment, without tying up heavy cranes. One set of side forms, together with three sets of soffit forms, permitted a weekly casting cycle and the raising of one roof girder per week. Cost was $8500 per girder in place, equivalent to about $2.50 per sq ft covered area, or $170 per cu yd of concrete.

STABILITY REQUIREMENTS DURING CONSTRUCTION

The fact that the roof girders were prestressed and lifted before they were held in position horizontally by the precast roof slabs called for special investigation as to their stability during the intermediate construction stages. This requirement proved to be harder to meet than the stability requirements for the completed structure.

It has been well established in the literature that there does not exist a buckling danger due to prestressing force in a straight or curved concrete column as long as the force induced in the prestressing members or member is located directly in the center of gravity of the concrete section. Any bending moment due to prestress resulting from a lateral deflection would be immediately counteracted by an equal and opposite bending moment due to the resistance of the taut prestressing unit. This phenomenon is sometimes misleadingly interpreted to mean that there is never any buckling danger in a prestressed member. Any outside force on a prestressed concrete member will produce the same buckling effect as if the structure were not prestressed. In fact, it may be reasoned that a prestressed column will buckle earlier than a nonprestressed column because of a lowering of the modulus of elasticity with the added compression due to pre-

stressing, as can be noted in a stress-strain curve for concrete. This apparent disadvantage with a prestressed compression member is, however, overcome by the fact that a prestressed concrete member behaving elastically will deflect less under lateral loads than a nonprestressed member, because it will not crack during the early stages of deflection like ordinary concrete does.

When investigating the buckling safety of a prestressed roof girder with vertically draped and grouted prestressing members, it can also be reasoned that there is no buckling effect in the vertical girder plane. Any buckling effect due to the vertical curvature of the cables will immediately be counteracted in this plane by an equal and opposite force exerted by the bending moments in the concrete.

The horizontal or lateral buckling effect due to a relative horizontal displacement of the position of the top and bottom flanges will, however, be similar to the torsional buckling in narrow I-beams. In narrow strip or I-beams such lateral buckling effect is practically independent of their great vertical stiffness and principally depends on the horizontal stiffness of the flanges and the torsional resistance of the total section.

Similarly with draped prestressing members, we can assume a great vertical stiffness which, however, is not contributing to the lateral stability. The statement found in the literature that a beam cannot buckle due to the prestressing force in the cables, merely applies to the action in the vertical plane. If the girder is narrow, it will still have a buckling tendency in the lateral direction.

Usually, as in the auditorium structure described, where the girder was cast on a stiff ground floor slab, the uplift forces from the draping of the cables are canceled by the dead weight of the girder.

If girder is cast on a resilient soffit base, the base may push upward, while the beam is deflecting upward during prestressing. In such case the dead weight of the beam may not cancel out the upward push due to the drape of the prestressing units. If at the same time, the supporting base offers no lateral resistance or permits an angular motion of the beam cross section, a relative lateral displacement of the beam flanges may occur during prestressing, causing stability problems.

A similar condition can also occur if during lifting a prestressed girder accidentally rests on an obstruction along the center of the span. This is a particular danger with prestressed beams made of a series of precast blocks and can cause an immediate failure by buckling or bending where narrow flanges are used.

In the auditorium girders, eccentricity due to the haunched shape of the beams also caused a concentrated upward force acting in the center of the beam, not counterbalanced by the dead load of the girder. The girder stability was calculated with the conventional formulas for lateral buckling of beams,[2,3] which are based on the torsional rigidity and horizontal flange stiffness of I-beams. Their application indicated a sufficient factor of safety during the various construction phases.

Literature on prestressed concrete hardly mentions the importance of stability

problems during erection. Leonhardt* in his recent book enumerates those problems and promises further data in a future edition. An early discussion is highly desirable.

STABILITY REQUIREMENTS FOR COMPLETED STRUCTURES

The ACI Code requires (Sec. 704) that the clear distance between lateral supports shall not exceed 32 times the least width of the compression flange. Had the roof slab been placed at the top along the compression flange, this ruling would obviously not apply for the final building condition, since the slab would offer a continuous lateral support. The principal question raised was: what lateral stiffness, if any, does a roof slab offer the girder when it is not located directly at the level of the compression flange?

The chosen economical width for the top flange was 3 ft 10 in. and the clear distance between column faces was 138 ft, or 36 times the flange width. The easiest solution would have been to widen the top flange 4 in. The code requirement would then be met for the case that no roof slab is available for lateral stiffness, but girder weight would increase 4 percent.

The code, however, merely states a ruling for required stiffness between "lateral supports." Certainly a slab located between top and bottom flange represents lateral support. Many long barrel structures have been built in this country with narrow supporting girders projecting upward attached to the thin shell. The writer has checked the lateral stability of a number of these and found it satisfactory. If a 3-in. moderately curved shell slab at the underside of a girder provides lateral stability, why should not a stiff-ribbed precast slab be expected to provide lateral stability as long as the slab panels are properly connected to the lower flange of the girder?

It is necessary to check not only the elastic support offered against buckling by the roof slab but also whether or not the web of the girder has the necessary strength to transmit the lateral stiffening action of the roof slab from its connection at the bottom flange to the top flange. Vertical stiffening ribs had been provided at about 22-ft spacing, so that this small distance might have been regarded as the spacing of lateral supports.

Actually the elastic stability for the top flange in the horizontal direction was calculated with the problem of beams on elastic foundation subjected to horizontal thrust equal to the compressive force in the top flange.[1,2] A so-called foundation modulus k was obtained by applying a unit live load horizontally along the top flange and calculating the resulting horizontal deflection, considering the elastic restraint of vertical flange into a sinuously deflected roof slab. It was found that the calculated factor of safety for the regular Euler buckling load of the top flange, assuming hinges at both ends of the 138-ft clear span, was only 1.37. A similar factor of safety considering the elastic lateral support of the roof slab was several hundred times this amount.

*Leonhardt, F., *Spannbeton fur die Praxis*, Wilhelm Ernst & Sohn, Berlin, 1955, 472 pp. See pp. 375, 376.

The ACI Code nowhere specifies outright the minimum factor of safety for buckling in a concrete structure. Sec. 1107 only permits compression members with a maximum length of 20 times the least lateral dimension, and Sec. 704 permits a beam span of 32 times the flange width. ACI Committee 312—Plain and Reinforced Concrete Arches—is recommending a design procedure based on ultimate strength.

The roof girder for the Springfield auditorium is 7 ft deep at the center and 5 ft 6 in. deep at the ends. Therefore the center of gravity of the girder is higher at the center than at the ends. This produces an arch action when considering the restraint into the columns. Is it therefore an arch? Clearly whatever reasoning we apply to the stability of an arch should tie into the requirements for stability of a frame and finally to the stability of a simple beam. It seems therefore that proof of a minimum factor of safety for buckling in all concrete structures to be calculated with the elastic theory should be a basic code requirement.

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For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Introductory paper of a four-part series on lightweight concrete

Title No. 53-20

Properties of Lightweight Aggregates and Lightweight Concretes*

By G. W. WASHA†

SYNOPSIS

Presents condensed history of use of lightweight aggregate. Lightweight concrete is divided into: cellular or foam concrete; no-fines or "popcorn" concrete; and lightweight-aggregate concrete. Properties of lightweight concrete are summarized. Lightweight aggregates, both mineral and vegetable, are enumerated, and the desirable and actual properties of lightweight mineral aggregates are discussed in some detail.

HISTORY

Lightweight aggregate was probably first used by the Romans who embedded large pieces of pumice in the walls and domes of some of their larger buildings. From the fall of the Roman empire until the present century only small amounts of lightweight aggregates were used. The Germans started using slag in 1822, and slag as concrete aggregate was first used in the United States in 1890. Cinders from coal-burning furnaces were used early in this century in industrial areas of the United States. Millions of cubic yards of sand-cinder concrete have been used, for example, in New York City. In 1918 the granting of the Haydite patent covering the production of bloated clays and shales brought another lightweight aggregate into the American market. During the past 20 years other materials such as pumice, vermiculite, and perlite have been widely used. Cellular or foam concretes have been developed and have had their greatest use in Europe. Patents relating to processes used today were obtained as early as 1914, and many patents in this field were granted from 1923 to 1929.

Another important factor that has contributed to the increasing use of lightweight aggregates goes back to the latter part of the nineteenth century. At that

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time a radical change in building design was brought about by the introduction of structural steel and structural concrete. The old method of designing buildings to carry loads by means of heavy load-bearing walls was discarded in favor of a design method in which the load was carried by a framework of beams and columns. Lightweight aggregates and the new design method made possible the construction of skyscrapers and long-span bridges. While the saving in wall and floor weight was extremely important, it was not, however, the only important problem. Thin walls created heat insulation problems, and lightweight aggregates also helped to solve this problem.

TYPES OF LIGHTWEIGHT CONCRETE

Lightweight concrete can be produced by the use of air or lightweight aggregates or combinations of both. Table 1 outlines the more important methods now used to produce lightweight concrete.

TABLE 1—TYPES OF LIGHTWEIGHT CONCRETE

Type	Cellular or foam concrete	No-fines or "popcorn" concrete	Lightweight aggregate concrete
Lightweight caused primarily by	Air	Air and/or specially graded no-fines aggregate (either lightweight or dense aggregate)*	Lightweight aggregate
Typical mix ingredients	*Moist cured* Portland cement, water, foaming agent—may also contain siliceous sand, silica, fly ash *Autoclaved* Portland cement, water, foaming agent, lime, silica, fly ash, expanded shale or slate dust, volcanic ash, pumice dust	*Moist cured* Portland cement, water, foaming agent, lightweight coarse aggregate, siliceous pea gravel or Portland cement, water, specially graded lightweight coarse aggregate *Autoclaved* Lime, silica, water, foaming agent, lightweight coarse aggregate	Comparable to conventional concrete ingredients except for use of lightweight vegetable or mineral aggregate

*See Forlani, Roberto, "No-Fines Concrete," ACI JOURNAL, Feb. 1950, *Proc.* V. 46, p. 477, and Valore, Rudolph C., and Green, William C., "Air Replaces Sand in 'No-Fines' Concrete," ACI JOURNAL, June 1951, *Proc.* V. 47, p. 833. Both references treat no-fines concrete made with ordinary-density aggregates.

Cellular or foam concrete

When a large amount of air is incorporated to reduce weight the resultant product is frequently referred to as foam or cellular concrete. This type of concrete is usually made of portland cement, water, and foaming agent, and may contain lime, silica, fly ash, expanded shale or slate dust, volcanic ash, or pumice dust. Cellular concrete contains a large amount of entrained air or gas, usually exceeding 25 percent, in the form of small bubbles which create a texture similar in appearance to sponge rubber; it is either moist cured or autoclaved. Valore* has discussed in detail the methods of manufacture and uses of this material.

Properly made, foam concrete has satisfactory strength combined with lightweight and low thermal conductivity. It can be produced in a wide range of densities and properties which may be varied to suit specific requirements. It can be easily molded to any desired shape, and the lighter types can be worked with woodworking tools. It is reasonably durable, noninflammable, and has good resistance to fire. With large-scale production its cost per unit volume is comparable with that of ordinary concrete. The chief disadvantages are its high setting and drying shrinkage and the dimensional changes arising from variations in moisture content. These disadvantages can be partly overcome by high-pressure steam curing. The lower and medium density grades have comparatively low resistance to abrasion and mechanical damage and should be protected. The production of uniform products with the desired properties requires considerable technical skill and experience and close control of the manufacturing process.

No-fines or "popcorn" concrete

In another type of lightweight concrete, air and either dense or lightweight coarse aggregates are used to make a product known as "no-fines" or "popcorn" concrete, usually having an air content between 20 and 40 percent. These high air contents are obtained by the use of foaming agents or by the use of specially graded coarse aggregates.

Lightweight-aggregate concrete

A third method of making lightweight concrete involves the use of lightweight aggregates. This method is by far the most important currently in use in the United States. Proper control of the manufacturing process, type and grading of aggregate, mix proportions, and curing method results in a wide variety of uses of lightweight concretes, made principally with mineral aggregates.

As shown in Table 2, lightweight aggregates may be considered under two headings—vegetable and mineral. The vegetable aggregates, with the possible exception of sawdust, have not been as widely used as mineral aggregates. Studies have been conducted on over 200 different organic materials, and test results on some of them have been considered encouraging. Sawdust especially has been used to some extent in this country, and has been extensively used in England and on the Continent to make such products as precast units for portable housing, flooring,

*Valore, Rudolph C., Jr., "Cellular Concretes," ACI JOURNAL, May 1954 and June 1954, *Proc.* V. 50, pp. 773-796 and 817-836.

TABLE 2—LIGHTWEIGHT AGGREGATES

Vegetable	Mineral	
Seaweed	*By-product*	*Expanded*
	Cinders	Slag, clay,
Straw		shale, perlite,
	Natural	vermiculite
Cork	Pumice	
	Scoria	*Sintered*
Sawdust	Tuff	Clay, shale,
		fly ash, diato-
Mineralized sawdust		mite, colliery
		waste

subflooring, precast building units, and cement lumber. It has excellent insulating value and light weight, but its use should be limited to situations where high strength is not a requisite and where moisture is either not present or is largely prevented from entering the concrete by some protective method.

PROPERTIES OF LIGHTWEIGHT CONCRETE

Table 3 summarizes the more significant properties of various important types of lightweight concrete. The values for each property and for each type of concrete represent only the range in which most of the values are *normally* found. No attempt has been made to present extreme ranges, and the decisions regarding inclusion or exclusion of values have necessarily been arbitrary. The range in values is usually large because of the large possible variations in type and properties of raw materials, mix proportions, and manufacturing and curing methods.

On the basis of data in Table 3, the following statements may be made:

1. In comparison with conventional concrete which weighs about 150 lb per cu ft, lightweight concrete varies between 20 lb per cu ft for light insulating concrete to 115 lb per cu ft for structural concrete.

2. The strength of lightweight concrete varies from low values for insulating concrete to values for structural concrete that compare favorably with those for excellent grades of conventional concrete.

3. The modulus of elasticity of lightweight concrete varies from about 30,000 psi for insulating concrete to 2,500,000 psi for structural concrete. The modulus of elasticity of lightweight structural concrete is usually about one-half to two-thirds that of conventional concrete of the same strength.

4. Water absorption by volume of lightweight concrete varies from about 5 percent for good grade structural concrete to about 50 percent for insulating concrete.

5. Drying shrinkage of lightweight concrete is variable, usually ranging between 0.01 to 0.50 percent. The effectiveness of autoclave curing in comparison with moist curing in reducing drying shrinkage is shown for cellular concrete.

6. The coefficient of thermal expansion for lightweight concrete is usually about the same as that of conventional concrete, except that the value for lightweight masonry unit concrete is usually smaller.

TABLE 3—PROPERTIES OF LIGHTWEIGHT CONCRETE

Property	Cellular concrete	Concrete made with lightweight mineral aggregates		
		Insulating	Masonry unit	Structural
Dry specific weight, lb per cu ft	20–60	20–80	65–100	65–115
Compressive strength, psi	100–2000	50–1000	1200–3000	1000–5000
Modulus of rupture, psi	50–500	15–150	200–450	150–500
Modulus of elasticity, psi × 10^6	70–500	30–500	500–1500	400–2500
Water absorption by volume	20–45 percent	15–50 percent	10–18 lb per cu ft	5–30 percent
Drying shrinkage, percent	0.01–0.15* 0.05–0.50†	0.05–0.50	0.02–0.14	0.04–0.15
Coefficient of thermal expansion per deg F × 10^6	4.5–7.0	3.0–7.0	2.0–4.5	4.5–6.0
Thermal conductivity, K	0.5–2.0	0.5–2.0	1.5–4.0	1.5–4.0

*Autoclave curing.
†Moist curing.

7. The thermal conductivity of lightweight concrete is less than that of conventional concrete. Values for insulating and for cellular concrete, 0.5–2.0 Btu per sq ft per hr per deg F, are especially notable.

LIGHTWEIGHT MINERAL AGGREGATES

Lightweight mineral aggregates are produced from natural deposits of pumice, scoria, volcanic cinders, tuff, and diatomite; from industrial cinders; by the application of heat to expand clay, shale, slate, diatomaceous shale, perlite, obsidian, and vermiculite; and by expansion of blast-furnace slag by cooling under more or less controlled conditions. These aggregates and methods of manufacture are described in some detail by Price and Cordon.*

The desirable properties of lightweight mineral aggregates are summarized below. Depending on the specific job, certain characteristics may be of much greater importance than others.

1. They should be uniform in composition and properties.
2. They should be suitably graded for their intended use, and the desired grading should be maintained.
3. They should have a low specific weight to provide a worthwhile saving in the weight of the structure and the desired special properties associated with light weight, such as high thermal insulation.
4. They should have a large number of small, well dispersed internal voids, but should have a minimum number of large external voids that have to be filled with mortar or paste.
5. Individual pieces should have adequate strength and should be firm and hard enough to withstand handling and mixing without size breakdown.
6. The particles must bond well with the cement paste and be inert chemically with respect to reactions with both cement and reinforcing steel.
7. They should have good resistance to weathering, moisture, insects, and fungi.

*Price, Walter H., and Cordon, William A., "Tests of Lightweight-Aggregate Concrete Designed for Monolithic Construction," ACI JOURNAL, Apr. 1949, *Proc.* V. 45, pp. 581-600.

TABLE 4—GRADING AND SPECIFIC WEIGHT REQUIREMENTS FOR LIGHTWEIGHT MINERAL AGGREGATES*

Aggregate type	Type of concrete	Grading of combined fine and coarse aggregates								Dry loose weight, lb per cu ft	
		Size designation	Percent by weight passing sieves with square openings								
			¾ in.	½ in.	⅜ in.	No. 4	No. 8	No. 16	No. 50	No. 100	
(a) Expanded or sintered products as blast furnace slag, slate, diatomite or fly ash; (b) Processed natural minerals as pumice, scoria, tuff; (c) Cinders†	Structural, masonry units, and insulating	½ in.–0	100	95–100		50–80			5–20	2–15	Maximum Fine—70 Coarse—55 F+C—65
		⅜ in.–0		100	90–100	65–90	35–65		10–25	5–15	
											Min. Max.
Perlite‡	Insulating					100	85–100	40–85	5–25	0–10	7½ 12
Vermiculite‡	Insulating				100	98–100	60–100	40–85	2–20	0–10	6 10

*ASTM specifications C 331-53T, C 330-53T, C 332-54T.
†Cinders for concrete masonry units only.
‡Requirement on the No. 30 sieve: perlite 20–60 percent; vermiculite 5–45 percent.

TABLE 5—PROPERTIES OF LIGHTWEIGHT MINERAL AGGREGATES*

Aggregate	Specific weight, lb. per cu ft.	Bulk specific gravity	Absorption, percent by weight	Crushing strength at 2-in. compaction, psi
Exfoliated vermiculite	6–12	0.9–1.3	20–35	35–45
Expanded perlite	4–16	0.7–1.1	10–50	60–400
Expanded slag	30–70	1.2–2.4	5–25	150–2000
Expanded shale, clay, slate	35–75	1.1–2.2	5–20	2000–16,000
Sintered fly ash	40–60	1.7–2.1	8–15	1000 ±
Pumice	25–60	1.0–1.7	8–50	1000–2000

*Based largely on information provided in "Lightweight Aggregate Concretes" issued in August, 1949, by Housing and Home Finance Agency.

Lightweight aggregates of a given type are apt to vary considerably because of differences in source of raw material and method of manufacture. This is of considerable importance because changes in the aggregates will obviously be reflected in the concrete made with them unless special precautions are taken. The tendency of some lightweight aggregates to produce harsh, segregated concrete must be overcome by proper proportioning of mix, or by additions of air-entraining or other workability agents. Lightweight mineral aggregates generally have higher absorption values and lower thermal conductivity, compressive strength, and unit weight than the conventional aggregates such as natural sand, gravel, stone, and air-cooled slag. With the exception of cinders, they are also likely to cost more.

For most uses lightweight aggregates should be free from harmful amounts of materials reactive with alkalies in cement, unhydrated pieces of hard burned lime or magnesia, calcium sulfate or unstable iron compounds, organic impurities, unburned clay particles, or other foreign materials which adversely affect the appearance or desirable properties. Lightweight aggregates used for structural concrete should have an ignition loss not in excess of 5 percent, while those used for masonry units, with the exception of cinders, should have a loss no greater than 8 percent.

Lightweight aggregates to be used for masonry units or for structural concrete should have a proven record of satisfactory durability or should be required to pass appropriate durability tests.

Grading

Lightweight aggregates are usually not available in sizes greater than ¾ in., and frequently are available only in sizes less than ⅜ in. Grading and unit weight requirements of lightweight mineral aggregates are given in ASTM standards C 330-53T, Lightweight Aggregates for Structural Concrete; C 331-53T, Lightweight Aggregates for Concrete Masonry Units; and C 332-54T, Lightweight Aggregates for Insulating Concrete. Table 4 shows unit weight and grading for combined fine and coarse aggregates given in these standards, and grading for perlite and vermiculite to be used in the manufacture of insulating concrete.

To assure reasonable uniformity in grading, the fineness modulus of the aggregate may not differ by more than 7 percent from that of the sample submitted for acceptance. If it does not meet this requirement, the aggregate may be rejected unless it can be shown that the aggregate will produce concrete of the required characteristics. It is also required that the unit weight of successive shipments of the aggregate shall not differ by more than 10 percent from that of the sample submitted for acceptance tests.

Representative properties and their determination

Table 5 gives representative values of specific weight, bulk specific gravity, absorption, and crushing strength of various classes of lightweight mineral aggregates. It should be emphasized that these values are representative only and that they do not include all possibilities. Further, it should be realized that the spread in values may be due to variation in aggregate from different sources and/or to the

differences between the values for fine and coarse aggregate of a given type. Fine aggregates, for example, are usually heavier and absorb less water than coarse aggregates.

While variations in the method of filling and compacting the measure used to determine unit weight influence the results obtained, the values given in Table 5 may be considered as loose dry weights.

Bulk specific gravity and absorptions are not easily determined for lightweight aggregates because of their rough vesicular surfaces and because of the differences in the absorptive properties of the various sized grains. As far as possible the methods given in ASTM standards C 127-42 and C 128-42 are used. Where the standard procedures for determining absorption have been impracticable the Bureau of Reclamation has used a dilution method. The method consists essentially of adding a supersaturated aggregate sample to a calcium chloride solution and noting the dilution by means of a special hydrometer. The weight of the free water thus determined is then subtracted from the total water to give the weight of the absorbed water.

The crushing strength of lightweight aggregates is determined by placing under light compaction a 5-in. depth of graded aggregate in a steel cylinder having an internal diameter of 3 1/32 in. and an external diameter of 6 in. A piston 3 in. in diameter is then pushed down against the aggregate and simultaneous readings of load and depth of compaction are noted. Correlation between aggregate and concrete compressive strengths is usually possible only in a general way.

Additional aggregate properties are indirectly specified in terms of their performance in concrete. For example, lightweight structural and masonry unit concretes are limited by ASTM specifications to a maximum drying shrinkage of 0.10 percent. Thermal conductivity of insulating concrete made with lightweight aggregate is specified not to exceed 1.50 Btu per sq ft per in. of thickness per hr per deg F, for 28-day oven-dry concrete weighing less than 50 lb per cu ft. It cannot exceed 3.00 Btu per sq ft per in. of thickness per hr per deg F for concrete weighing up to 90 lb per cu ft.

For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-21

The second paper in a series of four on lightweight concrete

Structural Lightweight-Aggregate Concrete*

By RALPH W. KLUGE†

SYNOPSIS

Properties of structural lightweight-aggregate concrete are treated, including a discussion of the proportioning and control of concrete mixtures containing lightweight aggregate, some of the physical characteristics of plain concrete, as well as certain structural properties of reinforced lightweight concrete with comments on their relation to the design of structural elements. This is an attempt to gather together all of the related data available, both published and unpublished, and present this information as a guide in the use of lightweight aggregates for structural concrete.

INTRODUCTION

The information contained herein covers primarily the expanded shale- and clay-type aggregates, expanded slags, and pumice. It should not be construed that these represent the only lightweight aggregates suitable for structural concrete. They merely happen to be the only types for which a reasonable quantity of information was readily obtainable. Certainly the majority of lightweight aggregates in current use for structural concrete are represented by these groups. This report is confined strictly to lightweight *aggregates*. Some lightweight structural concretes include blends with natural sand to increase strength or to alter certain other properties. Such combinations represent, in a sense, special classes of lightweight concrete and were not considered within the scope of this report. Further, the information contained herein pertains to concretes having a plastic consistency when cast, as distinguished from pressed or tamped concrete used in masonry units.

Definition of lightweight structural concrete—For the purpose of this report, lightweight structural concrete is arbitrarily considered to be a material weighing

*Presented as a part of the work of ACI Committee 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete, at the ACI 51st annual convention, Milwaukee, Wis., Feb. 22, 1955. Title No. 53-21 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 4, Oct. 1956, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Feb. 1, 1957. Address 18263 W. McNichols Rd., Detroit 19, Mich.

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not more than 115 lb per cu ft and one developing a compressive strength not less than 1000 psi. Although some concretes of lower strength are included, they are presented to indicate a trend of certain relationships.

Sources of information—In addition to exploring considerable material in the technical literature pertaining to the subject, information was obtained from descriptive brochures available from the lightweight-aggregate industry and from commercial laboratory reports. Many unpublished results of tests conducted by aggregate-producer associations were made available for study. In all, some 85 references in the form of technical papers, articles, and abstracts were examined for pertinent information. A selected list of references used directly in the preparation of this report is appended.

PROPORTIONING AND CONTROL OF LIGHTWEIGHT-AGGREGATE MIXES

Aggregate gradation

Experience both in the laboratory and in the field generally indicates the desirability of proportioning lightweight aggregates to give a higher percentage of fine material than is usual with the conventional heavier aggregates. Such practice is understandable since many lightweight aggregates are sharp, angular, and sometimes rough in texture; on the other hand, too much fine material produces an uneconomical mix. The usual ratio of fine-to-total aggregate by loose dry volume lies between 50 and 60 percent depending upon the maximum size aggregate, the particle shape, the individual gradation of the fine and coarse sizes, and whether the concrete is to contain entrained air.

The American Society of Testing Materials in its C 130-42* specification for lightweight aggregates defines the limits of fine and of coarse material passing standard sieve sizes. The standard for fine is almost identical to that for natural sand, but the two are not comparable because the percentages passing are based on weight, and substantial differences in specific gravity of the various fractions of lightweight aggregate exist that do not occur in natural dense sands.

Such standards are primarily a guide for general use. The ultimate objective is to obtain a combined gradation of fine and coarse material which will produce a workable mix, economical in cement for the quality of concrete desired. It is generally considered, and there is no specific evidence to the contrary, that aggregate which gives a uniform gradation curve will usually produce the most satisfactory results.

Air entrainment

It is generally recognized that the most effective method of producing workable concrete without increasing the water requirement or the unit weight is through the use of entrained air. Air entrainment tends to reduce segregation, bleeding, and if anything, tends to reduce the water requirement. The effect of air entrain-

*C 130-42 has since been replaced by C 330-53T, C 331-53T, and C 332-54T.

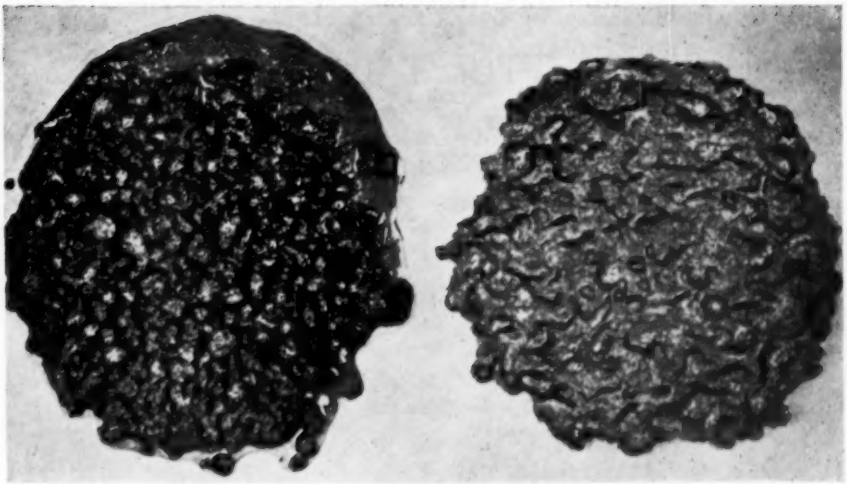

Fig. 1—Harsh mixtures with and without air entrainment

ment on the reduction of segregation is illustrated in Fig. 1, which shows two mixes containing identical proportions of cement, lightweight aggregate, and water; one contained entrained air and the other not. There exists a limit beyond which entrained air will cause a reduction in strength of lightweight concretes, but apparently that limit is somewhat above that for dense-aggregate concretes. Murlin and Willson[1] state ". . . that for a given volume of entrained air the reduction in strength is about half as much for small aggregate as for large aggregate. In other words, twice as much air can be entrained in small-aggregate concrete with the same loss of strength as for large-aggregate concrete." Neither the total quantity of air entrained which causes this reduction in strength nor the method of measuring the amount of air was indicated.

The optimum quantity of air is probably closely related to gradation and, as pointed out above, to the maximum size of aggregate. Therein lies a field for further study, for air bubbles may, in a sense, be considered as aggregate which promotes both workability and reduction in weight, and within certain limits without sacrifice in strength.

Murlin and Willson[1] suggest that about twice the amount of Vinsol resin solution (air-entraining agent) specified for heavy concrete be used, or about 1 to 2 oz per sack. Kluge, Sparks and Tuma[2] used as high as 0.05 percent neutralized Vinsol resin by weight of cement. This would amount to about ¾ oz per sack. The maximum used by Green and Watstein[3] was 0.04 percent, and this for the leaner mixes. The latter also reported air contents as high as 23 percent in expanded-shale and 28 percent in expanded-slag concretes; however, it should be pointed out that these were relatively weak concretes of 900 psi or less. For the stronger concretes this figure was nearer 10 percent. It is evident that the more rough, vesicular, and sharp the surfaces of the aggregates

are, the more air entrainment necessary. In view of the lack of definite information it would appear prudent perhaps to use as little entrained air as is necessary to provide the desired workability. However, some air entrainment is highly desirable from the standpoint of durability or resistance to freezing and thawing. This has been effectively demonstrated for both dense- as well as lightweight-aggregate concretes.[2,4]

Some methods of measuring entrained air are not entirely satisfactory when applied to lightweight-aggregate mixtures. The air-meter method, for example, measures not only entrained air but the air confined in the unfilled surface voids of crushed aggregates. The ASTM gravimetric method requires an accurate value of the aggregate's specific gravity, which is difficult if not impossible to determine accurately for many lightweight materials. The modified Indiana (or Ohio) method and the "rollmeter" methods, as developed by Menzel,[4] both of which are basically similar, are perhaps the most satisfactory methods for vesicular aggregates. The rollmeter equipment is reported to be in extensive use with lightweight-aggregate concrete.

Preparation of aggregates

The weight of opinion among those experienced with the mixing and placing of lightweight-aggregate concrete favors presoaking the aggregates. Unfortunately, there is no information available concerning the time required to soak the various aggregates to produce an absorption equal to that taking place during mixing and placing of concrete where the aggregates are initially dry. It has been argued that absorption of water from the mixed concrete tends to reduce the water-cement ratio and therefore should increase strength. This reasoning evidently assumes that most of the absorption takes place after mixing but before initial set of the concrete. It has also been pointed out that absorption after initial set might well affect the process of hydration, thus adversely affecting its strength.

Richart and Jensen[5] claimed greater concrete strength by using moist aggregates, but from a study of their data it would appear that this was due to higher cement contents for mixes cited, rather than to the damp aggregate. Duey[23] on the other hand reported increased compressive strength for initially dry aggregate. His data show a consistent decrease in 28-day strength with an increase in initially absorbed moisture. The range of strength was from approximately 4400 psi at zero moisture to about 3800 psi at 17 percent absorbed moisture. There was also a slight decrease in shrinkage at 180 days for initially dry aggregate.

Saturation of the aggregate is generally impractical and appears to be entirely unnecessary. Tuthill[24] reported that experience in the concrete ship program, 1942-45, where expanded shale- and clay-type aggregates were used, indicated that better control and better concrete (at least lighter weight) could be obtained without saturating the aggregates. Tests conducted in one shipyard were cited to show that compressive strengths increased slightly and unit weight decreased about 3 lb per cu ft (and remained so in wet storage) with use of dry rather than saturated coarse aggregates. It is presumed that "dry" meant reasonably dry, not completely dry. Whereas the procedure in the casting yards was, initially,

to saturate aggregates, on frequent occasions aggregate less than saturated was used before the program terminated.

It is probable that much of the benefit many have ascribed to wetting of aggregate before mixing was in the improvement in uniformity of gradation and greater freedom from separation that wetting would provide in materials containing wide ranges of sizes.

From a practical standpoint it is probably much easier to control consistency of the concrete if the aggregates are presoaked to some nominal moisture content, considering the peculiar absorptive characteristics of most lightweight aggregates, particularly of those which have been crushed in processing. Richart, for example, has shown that for crushed expanded shale the initial moisture content of the aggregate has considerable influence on the total moisture absorbed in a given period of time; further, the rate of absorption of fine material depends upon its gradation and reaches its absorptive capacity more quickly than the coarse.

Proportioning the mix

Probably the generally accepted water-cement ratio law for dense-aggregate concrete is equally applicable to any given lightweight-aggregate concrete, but the problem of determining the surface moisture available as mixing water from crushed presoaked aggregates is not simple. Attempts have been made to develop a simple and reliable procedure to determine the saturated surface-dry condition of various aggregates but with little success.[5,6] Furthermore, for the water-cement ratio law to be useful, numerical values expressing it would have to be established for each particular aggregate because of the wide differences in physical properties between different aggregates. It would therefore appear, considering the present state of our knowledge, that proportioning mixes or controlling concrete strengths by application of the water-cement ratio law, *per se,* is definitely impractical in the majority of cases.

Mix proportioning is fundamentally the same as for heavy concrete, except that generally proportions are not based on a numerical value of water-cement ratio unless such a value has been established for the particular aggregate under consideration, and the quantity of water available from the moistened aggregates for mixing has been determined with reasonable accuracy. It is strictly a trial and error procedure with a few guides to assist in the process.* The proportions of fine and coarse materials evidently depend upon individual gradations for these two sizes. Having sieve analyses for each of these materials, a combined gradation curve can readily be calculated for various proportions of the two sizes and a ratio selected which produce an over-all gradation comparable to those which have apparently proven satisfactory. The proportions of cement to aggregate for a given strength depend upon a number of factors such as the crushing strength of the aggregate, its gradation, and the water-cement ratio. The quantity of water in turn depends on consistency desired, the quantity of air entrained, or upon the nature and amount of any other admixtures used.

*Although suggested mix proportioning specifications are available from many aggregate producers, they are generally limited to concrete of a particular compressive strength.

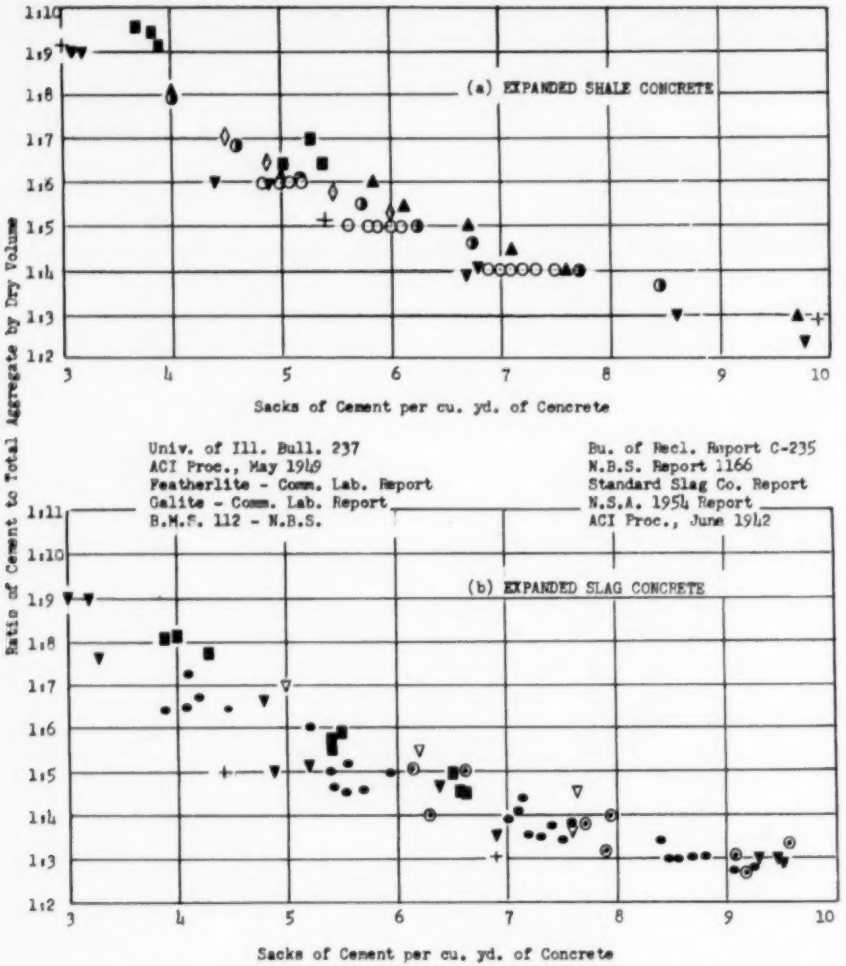

Fig. 2—Relation between cement content and ratio of cement to total aggregate for expanded-shale and expanded-slag concretes. Symbols same as in Fig. 5

Fig. 2 and 3 show a general relation between the proportion of cement to total aggregate and cement content for concretes containing aggregates of one of three general types. The data include all of the variables mentioned above which affect the yield and therefore the cement factor. Consequently, a wide spread occurs in the plotted values. Nevertheless, these data are of value as a guide for trial batching. The quantity of water depends on the consistency desired, once the proportions of cement, aggregate, and admixtures are established, and is determined by trial. Employing certain admixtures and/or air entrainment, concretes having slumps of 2 to 4 in. are reported to be placed regularly without difficulty.

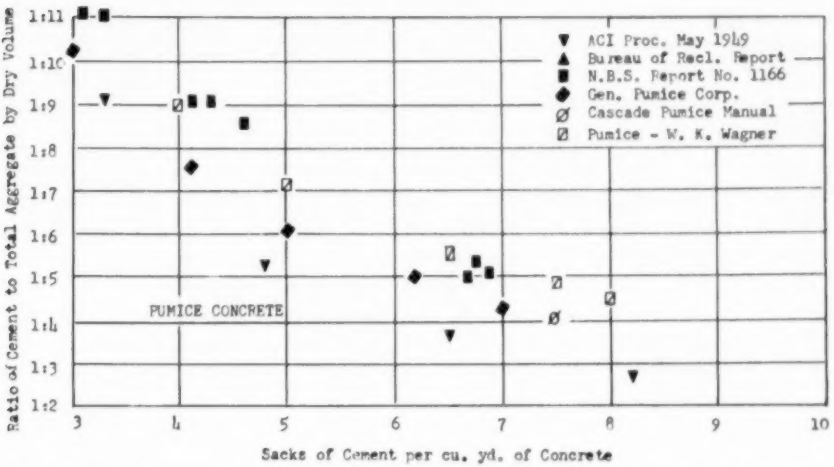

Fig. 3—Relation between cement content and ratio of cement to total aggregate for pumice concrete

Control of quality

It is of little value to carefully proportion a mix unless adequate control is exercised over subsequent manufacturing operations. Probably the most important element in the control of quality in lightweight-aggregate concretes is that of uniformity of gradation of aggregate from batch to batch. The specific gravity and absorptive properties of lightweight aggregates, particularly of the crushed variety, vary over a considerable range through the various fractions of fine material. Consequently, changes—even of a normally minor nature—may seriously affect the consistency, the water requirement, and the strength.

Wagner[7] illustrated the effect of varying the gradation fine pumice from the lower to the upper limit of gradation established by ASTM C 130-42* by calculating the variation in absorption and bulk specific gravity. He reported a change in specific gravity from 1.95 to 1.60 and absorption from 18.0 to 29.8 percent by such a change in gradation.

A factor which can inadvertently alter both gradation and yield, is a change in moisture content of the aggregate from time to time. The proportioning of the mix and subsequent batching should be based on the unit weights of moisture-free aggregate and frequent checks made on moisture content, or on unit weight of fresh concrete, so that frequent adjustments can be made in batch weights.

It is evident from what has been discussed that methods of storage and handling of aggregates also are important factors in maintaining uniformity of gradation. At least one lightweight concrete mixing plant uses silos for storage of aggregate to prevent segregation and to provide material of a constant moisture content.

*See footnote p. 384.

Assuming that uniformity of gradation is accomplished, there yet remains the matter of controlling water. Obviously, there would be no need for this if the moisture content of the aggregates remained constant. Laboratory and field experience indicates that the slump test is about as satisfactory a method of checking on water as anything yet suggested. An experienced mixer operator can usually estimate the water requirement fairly accurately from the results of such a test. On tilting drum mixers, where the mixing action may be observed, a fairly reliable method of ascertaining consistency is to observe the flow of concrete with respect to the mouth of the mixer while it is revolving.

An example of close control achieved under field conditions, in which the quantity of water was controlled by slump tests, is cited by Murlin and Willson.[1] They reported a strength range of 3993 to 4332 psi at 42 days from nine cylinders, presumably cast from random batches. The average compressive strength was 4044 psi and the design strength was 4000 psi. It should be mentioned that aggregates were prewet; but just how wet, or how much moisture was absorbed was not indicated, although the quantity of absorbed water was probably of little importance.

Mix control in such large and widely scattered operations as the concrete ship program[24] was apparently achieved without particular difficulty through the use of the slump test and unit weight checks. It is of interest also to note other methods of control exercised in this program to insure quality concrete. Although specific gravity tests of the aggregates were regularly made, batch weights (persumably of the aggregates) were adjusted on the basis of unit weight of fresh concrete. Also whole batches were checked for volume in calibrated containers to produce a concrete of the proper yield and cement content.

PROPERTIES OF PLAIN LIGHTWEIGHT CONCRETE

Compressive strength and its relation to cement content

The data of Fig. 4 and 5 are presented as a guide in the proportioning of concrete mixes as well as for general information regarding the approximate quantities of cement necessary to achieve certain strengths with these aggregates. Since these data include all of the variables which affect strength, a wide spread in plotted values is to be expected. The plotted data, however, were generally confined to mixes having a slump of 6 in. or less, and most were for slumps of 4 in. or less. It appears that for a given aggregate, substantial variations in cement content are possible for a given strength or, conversely, substantial variations in strength are possible for a given cement content. Since it is presumed that the crushing strength of a particular aggregate is constant in such instances, and since it is assumed further that the air content and consistency of the particular mix are reasonably constant, the differences appear to result from variations in gradation with concomitant differences in water required. It would therefore seem desirable, in proportioning mixes, to give careful con-

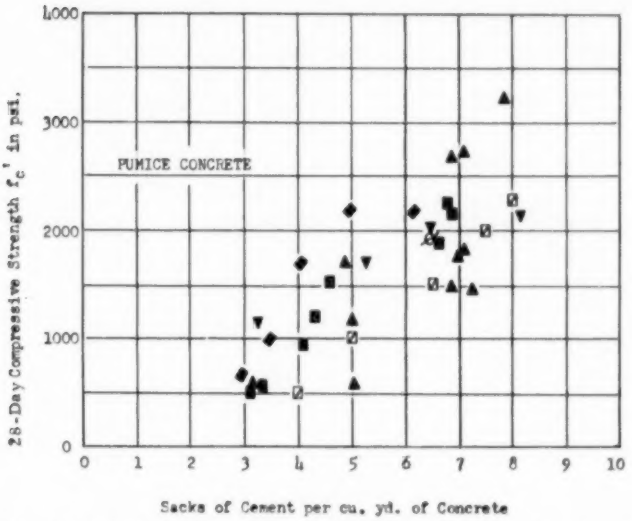

Fig. 4—Relation between compressive strength and cement content for pumice concrete

sideration to the proper gradation of aggregate, and maintaining the selected gradation in subsequent mixing operations.

Although all of the strength data given in this report are for standard curing at 28 days, there is considerable evidence to indicate that lightweight-aggregate concretes develop strength beyond this age at a much greater rate than does conventional concrete. There is the question, therefore, whether 28-day values fairly represent such concretes. It is not the intention here to discuss this question, but merely to call attention to it, since other factors as well as the age-strength relationship are involved.

Unit weight versus compressive strength

The unit weight of lightweight concrete depends upon the unit weight of the aggregates, the richness of the mix, and the quantity of air entrained. The latter has a rather important effect because of the relatively large percentages of air usually entrained in lightweight-aggregate concrete. In general, the heavier the aggregate the greater its crushing strength, a factor which influences the compressive strength of the concrete. However, crushing tests of lightweight aggregates which have been made and reported[4,6] show only a general correlation with the strengths of the concretes made therefrom. Aggregate gradation may also be an important element in the strength-weight relationship; therefore, it would appear possible to vary this ratio within certain limits by modifying the gradation, all other things being equal.

The range of concrete weights covered in this report is from approximately 55 to 110 lb per cu ft fresh, and from 50 to 109 lb per cu ft dry. With a few ex-

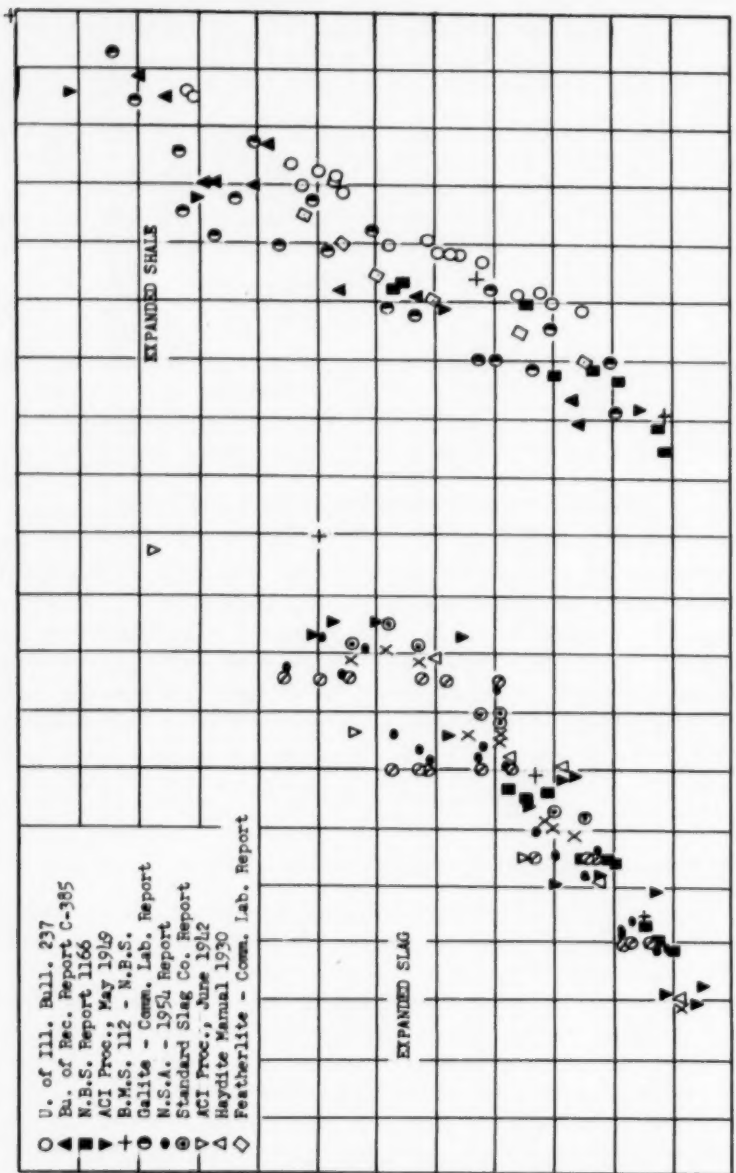

Fig. 5—Relation between compressive strength and cement content for expanded-slag and expanded-shale concretes

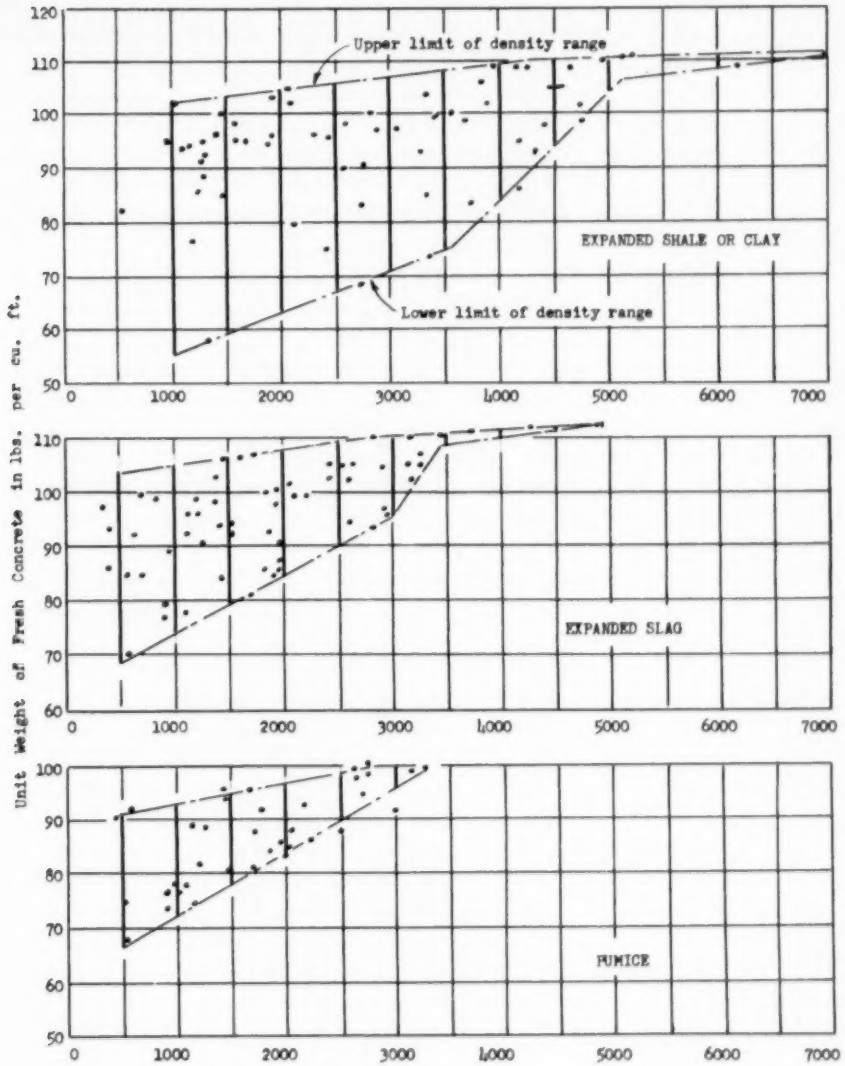

Fig. 6—Relation between density and compressive strength

ceptions, those weighing less than about 60 lb per cu ft dry developed compressive strengths of less than 1000 psi, so that for all practical purposes this seems to be the lower weight limit for structural concrete according to the initially assumed definition. Fig. 6 shows these ranges in unit weight for various compressive strengths and for three types of aggregate. As might be expected, the range in unit weight narrows as the compressive strength increases. The lighter-weight

aggregates and the mixes containing the larger quantities of entrained air, are generally represented in the lower limits of concrete weight. It is evident that larger percentages of cement are necessary with these aggregates and mixes to increase their strength proportionately as compared to the heavier and usually stronger aggregates represented in the upper limits of the chart. Since cement is the heaviest element in the mix, the unit weight of the concrete containing the lighter aggregates would be expected to increase more rapidly with strength. Economic considerations would, of course, impose a concrete strength limitation for these aggregates. This also is evident from the charts.

One expanded-shale concrete developed 7045 psi at 28 days with unit weights of 111 and 109 lb per cu ft, fresh and room dry, respectively. Type III, high-early-strength cement, was used with a cement content of 8.5 sacks per cu yd. Reported room-dry weights did not show a consistent relationship to fresh weights, undoubtedly because of the different conditions under which the concretes were permitted to dry. For expanded-shale concretes the dry-to-wet weight relationship varied between 88 percent for concretes less than 1000 psi to about 99 percent for the high strength concretes but, in general, was about 95 percent. The dry weights of expanded-slag concretes ranged from 85 to 98 percent of fresh weights with 90 percent a general value. Pumice concretes generally showed the greatest percentage reduction in weight due to drying to room temperature varying from 70 to 98 percent with about 85 percent a general value.

Elastic properties

Data pertaining to the elastic properties of lightweight-aggregate concretes are much less extensive than those covering compressive strengths. The latter property seems to have been given much greater attention than modulus of elasticity, and yet from the standpoint of deflection, the elastic modulus is most important. In the design of structural members it is involved in the value of the modular ratio n, but as indicated later, it is much less important in this respect than would at first appear.

The work of Richart and Jensen,[5] which represents the first and most extensive research yet reported on the structural properties of expanded-shale (Haydite) concrete, definitely shows this concrete to have an initial tangent elastic modulus of 50 to 55 percent that of sand-and-gravel concrete of equal strength. The heavy solid line in Fig. 7(c) represents very closely results of their tests, and being practically linear, it can be expressed by the following equation:

$$E = 1.5 \times 10^6 + 160(f_c - 1000)$$

The data on which this equation is based included more than 65 values, each the average of three tests. They represented a variety of mixes, consistencies, and strengths and were remarkably uniform and consistent. Rather than show the numerous individual values, a line is used to avoid confusion with other data shown in the figure. Values of E obtained from other sources for expanded-

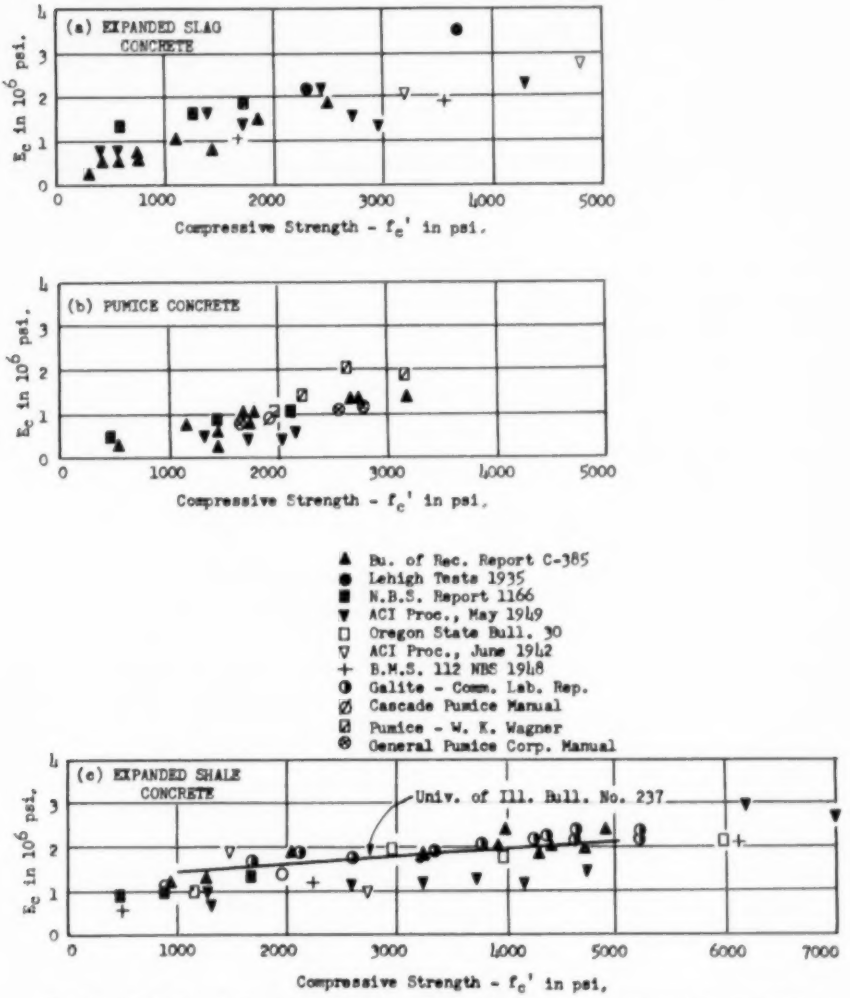
(a) EXPANDED SLAG CONCRETE
E_c in 10^6 psi.
Compressive Strength - f_c' in psi.
(b) PUMICE CONCRETE
E_c in 10^6 psi.
Compressive Strength - f_c' in psi.
Bu. of Rec. Report C-385
Lehigh Tests 1935
N.B.S. Report 1166
ACI Proc., May 1949
Oregon State Bull. 30
ACI Proc., June 1942
B.M.S. 112 NBS 1948
Calite - Comm. Lab. Rep.
Cascade Pumice Manual
Pumice - W. K. Wagner
General Pumice Corp. Manual
(c) EXPANDED SHALE CONCRETE
Univ. of Ill. Bull. No. 237
E_c in 10^6 psi.
Compressive Strength - f_c' in psi.

Fig. 7—Relation between modulus of elasticity and compressive strength

shale concrete are, in general, equal to or lower than those expressed by the above equation.

In Fig. 7(a) similar data are shown for expanded-slag concrete. Although the available data are not as extensive, it appears that the relationships between strength and elastic modulus do not differ materially from those of expanded-shale concrete. One exception is the unusually high value of E at a strength of about 3700 psi, obtained in the Lehigh tests.[8]

Even fewer data are reported for concrete containing 100 percent pumice aggregate. The available information is shown in Fig. 7(b) and is limited to con-

cretes of less than 3200 psi compressive strength. The highest values of E reported are, in general, lower than those for the other two types of aggregate. It is common practice to add varying amounts of natural sand to pumice concrete, which not only increases its strength but apparently substantially increases its elastic modulus as well.

STRUCTURAL PROPERTIES OF REINFORCED LIGHTWEIGHT CONCRETE

Bond

Of the few bonds tests reported, most refer to expanded-shale concretes. For other types of lightweight-aggregate concrete there is little information on bond. Unfortunately, test methods for bond have not been standardized until relatively recently and reported data appear to reflect the lack of standard procedure. Fig. 8(a) shows a wide range of values relating bond resistance to compressive strength for expanded-shale concrete. No two sets of data are closely comparable, in that they vary in the length of bar embedment, type of bar, and in end-slip observations. The University of Illinois tests and apparently the Oregon State College tests employed the old type of deformed bar now obsolete. The National Bureau of Standards and Georgia Lightweight Aggregate Co. tests used the ASTM A 305 bar.

It is almost impossible, under these circumstances, to make a fair comparison of these data with the test results obtained for conventional concrete. Richart and Jensen[5] showed a comparison with the results of a parallel series of tests of gravel concrete. Unfortunately, however, most of their gravel concretes were of higher compressive strengths than the all-Haydite concretes. If one were to extrapolate their data to make the comparison within the equivalent range of compressive strengths, the maximum bond values for the lightweight concrete would compare favorably with those of gravel concrete. Green and Watstein[3] state that ". . . the observed ratios of bond strength to compressive strength were generally in agreement with data reported by other observers for dense concrete." This statement presumably includes bond data for the expanded-slag and pumice concretes which were included in their study. It should be pointed out that the average maximum concrete strength in this study was under 2500 psi.

All of the data pertaining to the A 305 bar lie well above the 1951 ACI Code requirement.

In Fig. 8(b) similar data on bond resistance and compressive strength are shown for expanded-slag and pumice concretes. Only maximum values are shown. Except for the data obtained from the National Bureau of Standards *Report* No. 1166, it is not known what type of bar was used to obtain the other values reported. For concrete of 2000 psi or less where comparisons can validly be made, the bond values compare favorably with those for expanded shale, indicated by the solid rectangular symbols in Fig. 8(a). In general, the bond values for these concretes are also well above those permitted by the 1951 ACI Building Code.

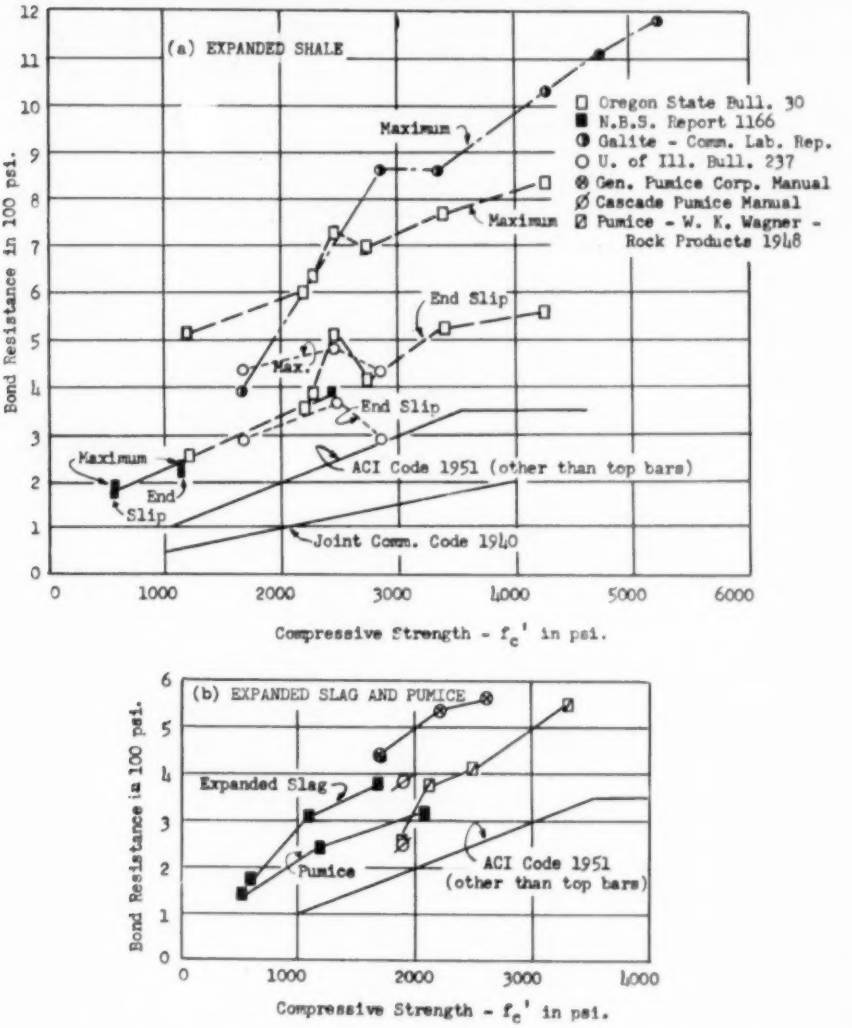

Fig. 8—Relation between bond strength and compressive strength

Shear

Shearing strength data for expanded-shale, expanded-slag, and pumice concretes are shown in Fig. 9 where they are plotted against the compressive strength of corresponding concretes as determined from 6 x 12-in. cylinders. Results reported in the University of Illinois Engineering Experiment Station *Bulletin* No. 237 are as much as 50 percent greater than those reported by others for the same type of concrete. Whether this is due to the differences in proper-

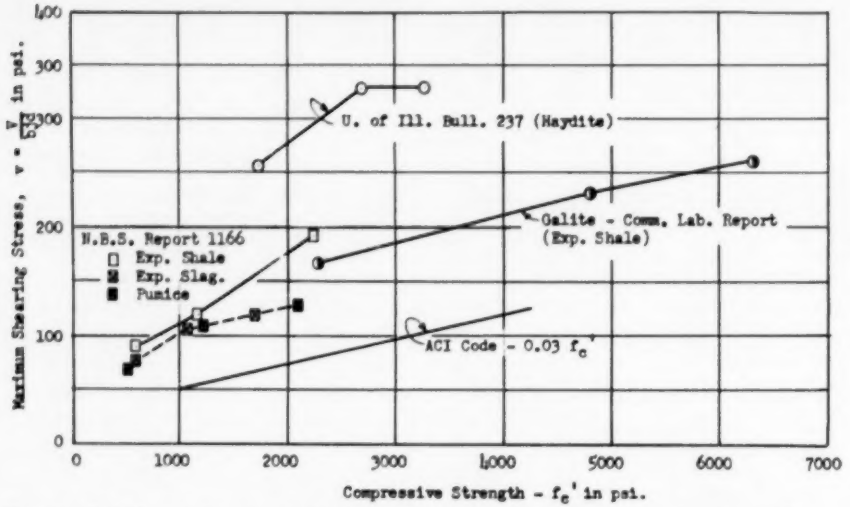

Fig. 9—Relation between maximum shearing stress and compressive strength

ties of the aggregates, shape of beam section, span to depth ratio, or air entrainment is not known. It appears that some factor or combination of factors may affect shear strength substantially. It may be noted that the tests at the University of Illinois were conducted with concrete containing no air-entraining admixture, whereas other tests involved such admixtures in varying amounts.

It appears that for the higher compressive strengths expanded-shale concretes develop greater shear resistance than do expanded-slag or pumice concrete. Both of the latter exhibit almost identical shear-strength characteristics up to about 2000 psi compressive strengths. Evidently these lightweight concretes are capable of developing shear strengths in some instances much greater than the ACI Code value of 0.03 f_c'.

Abnormally high shear-strength values have been given in some of the promotional literature from commercial testing laboratories. It is suspected that these values do not represent diagonal tensile strengths, but rather are data from attempted pure shear tests. One laboratory report definitely stated that the tests were conducted on small unreinforced beams in "double shear."

DESIGN ASPECTS OF REINFORCED LIGHTWEIGHT CONCRETE

Beams and girders

The design of lightweight-aggregate concrete flexural members would appear to follow in general the same procedure as applied to conventional heavy concrete. This statement is necessarily qualified in that there are relatively few data present in the literature relative to the behavior of reinforced lightweight concrete in flexure. Perhaps the major design question is the matter of a proper value for the modular ratio n to be applied to these concretes.

The 1940 Joint Committee* report requires that n be doubled for lightweight concrete, thus implying that the elastic modulus for this material is one-half that of conventional concrete of equal strengths. However, the value of n need not play an important role in the design of flexural members unless the designer wishes to work the concrete to the full allowable value, which usually is not the most judicious design procedure with lightweight concrete. Because of the lower values of E for these materials, deflection is probably the major problem. Theoretically, for balanced design the larger the value of n the less the effective depth of the section, which only aggravates the deflection problem. Therefore, deeper sections than those based on the increased value of n are indicated as a means of counteracting greater deflections. For depths greater than the theoretical, n loses its significance except insofar as one may wish to calculate the exact theoretical value of k and j. For all practical purposes the value of j may safely be assumed not to change for depths greater than that indicated for balanced design. Concrete stress under this condition need not be calculated since it is always less than the allowable working stress. T-beams almost always are controlled by the depth necessary at the supports, and the approximate value of j (namely $0.4t/d$) is entirely adequate for the determination of the steel area in a T-section subjected to positive moment. Concrete stress is normally of no concern when the flange is in compression.

Increasing the depth of section obviously causes reduction in the reinforcement required. This can be an important factor from the standpoint of economy.

Columns

Because of the relatively low elastic modulus of lightweight concretes, the behavior of columns constructed from these materials would be expected to differ in many respects from those of conventional heavy concrete. In the elastic range, stresses in the reinforcement would likely be somewhat higher due to the relatively larger value of the modular ratio. However, such stresses are meaningless in predicting the maximum load a column may carry, and it is on ultimate load that modern column design is based.

The only known experimental data pertaining to the behavior of lightweight-concrete columns is that reported by Richart and Jensen.[5] They tested ten all-Haydite-concrete columns, 8 in. in diameter and 5 ft long, with approximately 1.5 and 4 percent of longitudinal steel. Four of their columns were tied, four contained 1.2 percent spiral steel, and two were completely unreinforced. The test results were compared with those from an equal number of gravel-concrete columns similarly reinforced. Comparison of ultimate strengths based on equivalent concrete strengths showed that the Haydite columns were only slightly less strong than gravel-concrete columns regardless of the value of the modulus of elasticity. Thus, n had little or no bearing upon the strength. Stress-strain curves, however, definitely reflected the lower elastic modulus of the lightweight concrete. Total deformation of the Haydite columns was considerably greater than for the gravel concrete.

*The 1940 Joint Committee Report has been discontinued, effective August, 1956.

The ratio of plain-column strength to cylinder strength was higher for Haydite concrete than gravel, being 0.88 as compared with 0.82 for the latter. It thus appears that the ACI Code formula for axially loaded tied or spiral columns is equally applicable for expanded-shale concrete compression members, axially loaded.

SUMMARY

Based on the information available for study, the following general observations may be made regarding lightweight-aggregate concrete developing not less than 1000 psi, defined for the purpose of this report as structural concrete.

1. The proportioning of lightweight concrete mixtures does not differ fundamentally from that of conventional heavy concrete. However since the unit weight and crushing strengths of lightweight aggregates vary over a large range, the proportioning of a mix is a separate and distinct problem for each aggregate. The water-cement ratio law appears equally applicable to these concretes, but its application assumes a reasonably accurate knowledge of the quantity of water for mixing which is available from the presoaked aggregate. Presoaking or prewetting of the aggregate appears to be the preferred treatment.

2. Control of quality in the field, although achieved in some instances without apparent difficulty, may be a problem because of the peculiar absorptive properties of most lightweight aggregates. Also, fine aggregate gradation has a relatively greater influence on concrete yield and water requirement than is the case for conventional dense aggregate. Therefore closer control is required with respect to uniformity of aggregate from batch to batch.

3. Although the elastic modulus of lightweight concrete is somewhat lower than for conventional concrete, the mechanics of designing most structural members is affected little if any by the higher value of the modular ratio. Deflection, however, is a problem which must be considered.

4. Reported values of bond strength between reinforcement and lightweight-aggregate concrete are quite variable, due in large measure to lack of a standard test procedure. In some cases the reported data seem to be comparable with values obtained for conventional concrete. In all cases the values reported from pull-out tests for the modern-type reinforcing bar (A 305) are 50 percent or more above that permitted by the 1951 ACI Building Code.

5. There are few data available on the shearing (diagonal tension) resistance of lightweight-aggregate concrete. The values that have been reported indicate that the concretes covered in this report were capable of developing strengths from 60 to 300 percent above those permitted by the 1951 ACI Building Code.

It should be emphasized again that many of these observations are based upon rather meager information. It is unfortunate that there are so few specific technical data available which are useful to the designer of structural members of lightweight concrete. It is quite natural that compressive strength data, being the easiest and least expensive to obtain, are in the greatest quantity; but the knowledge of compressive strength alone does not insure a safe and economical structure.

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For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-22

Helicoidal Staircases of Reinforced Concrete*

By VICTOR R. BERGMAN†

Fig. 1—Helicoidal staircase at 1407 Broadway, New York, designed by Severud-Elstad-Krueger

SYNOPSIS

The analysis of a helicoidal staircase involves torsional moments as well as bending moments and shears and is consequently somewhat more difficult than that of a straight staircase. However, if the staircase is reduced to its horizontal projection, the problem becomes quite tractable since it resolves itself into that of a fixed-end, curved beam loaded normal to its plane of curvature.

This paper covers determination of the bending moment, torsional shearing stresses, shear reinforcement, longitudinal reinforcement, and design at the supports.

Recent years have witnessed the construction in reinforced concrete of curved staircases supported only at top and bottom (Fig. 1).[1,2] Since the curves of these staircases usually appear as circular arcs in plan projection, the description "helicoidal" is properly applied to such structures.

The analysis of such a staircase involves torsional moments as well as bending moments and shears and is consequently somewhat more difficult than that of a straight staircase. However, if we permit ourselves the same stratagem with the curved staircase that we employ with the straight—that of reducing the staircase to its horizontal projection—the problem becomes quite tractable, since it resolves itself into that of a fixed-end, curved beam loaded normal to its plane of curvature.

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At any cross section of the slab there will exist, in general, a bending moment, a torsional (twisting) moment, and a vertical shear. However, under the customary design condition of uniform loading covering the entire span, symmetry of loading and structure dictates that the torsional moment and the vertical shear must vanish at the midspan cross section. This leaves only the determination of the bending moment at that section to render the remainder of the analysis statically determinate insofar as the calculation of bending and torsional moments and shears is concerned.

DESIGN PROCEDURE

Application of the method of least work[3,4] to a fixed-end, curved beam of constant centerline radius R, leads to the following expression for M_c, the midspan bending moment:

$$M_c = wR^2\left[\frac{2(K+1)\sin\theta - 2K\theta\cos\theta}{(K+1)\theta - (K-1)\sin\theta\cos\theta} - 1\right]$$

in which

θ = one-half the total central angle (Fig. 2)
$K = EI/GJ$, the ratio of flexural stiffness to torsional stiffness
w = total load (DL + LL) per foot of span, measured along the longitudinal centerline of the plan projection

This formula for M_c may conveniently be written:

$$M_c = wR^2(U - 1)$$

where U represents the fractional part of the bracketed expression. U is seen to be a function of only K and θ. For any particular slab cross section, K may be

TABLE 1—VALUES OF K FOR RECTANGULAR CROSS SECTIONS OF WIDTH b AND DEPTH h*

$b \geq h$		$h \geq b$	
b/h	K	h/b	K
1.0	1.39	1.0	1.39
1.2	1.18	1.2	1.70
1.5	1.00	1.5	2.25
2.0	0.86	2.0	3.42
2.5	0.79	2.5	4.92
3.0	0.75	3.0	6.70
4.0	0.70	4.0	11.15
5.0	0.68	5.0	16.85
7.0	0.65	...	.....
10.0	0.63	...	.....
∞	0.59	...	.....

*Assumption: $E/G = 2.35$.

obtained from Table 1. The U corresponding to this K and to a given θ may then be read from the curves of Fig. 2, or else computed.

With U known, M_c is, of course, readily determined. So also are M_A and T_A,

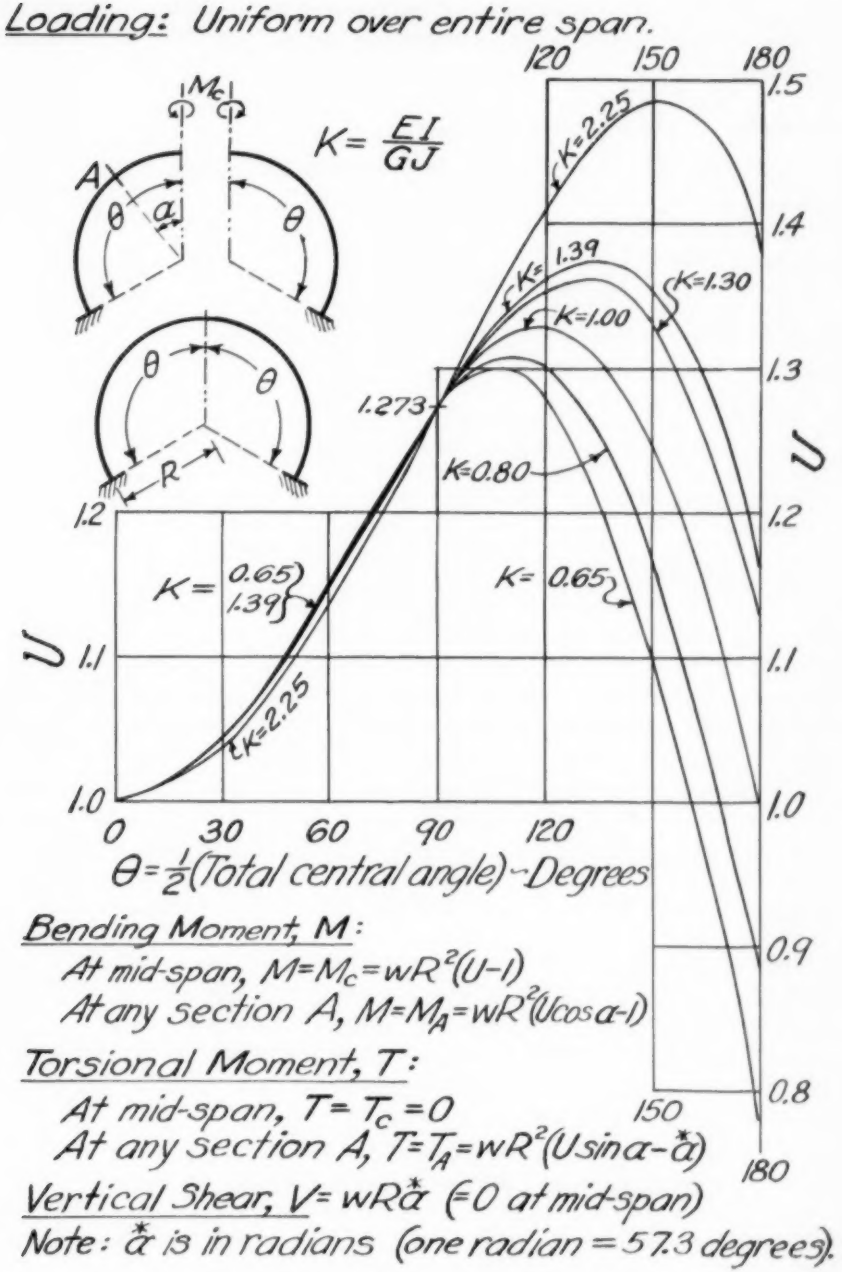

Fig. 2—**U** versus θ for various **K** values

respectively the bending moment and the torsional moment at any cross section A, located at an angular distance of α from midspan, by means of expressions given in Fig. 2.

Example

To illustrate the procedure, the following numerical example is offered.

Assume

Width of slab = 6 ft
Inner radius of slab = 6 ft
Average normal thickness of slab = 10 in.
Total load (DL + LL) = 275 psf of horizontal projection
Total central angle = $2\theta = 140°$

Solution

$w = 6.0 \times 275 = 1650$ lb per ft
Centerline radius = $R = 6.0 + 6.0/2 = 9.0$ ft
Ratio of slab width to thickness = $b/h = 6 \times 12/10 = 7.2$
For $b/h = 7.2$, $K = 0.65$, as read from Table 1
Fig. 2, entered with $K = 0.65$ and $\theta = 70°$, yields $U = 1.20$
Then,

$$M_c = wR^2(U - 1) = 1650 \times 9^2(1.20 - 1) = 26{,}700 \text{ ft-lb}$$

At the support, $\alpha = \theta = 70° = 1.222$ radians; $\sin \alpha = 0.940$; $\cos \alpha = 0.342$.

$$M_s = wR^2(U \cos \alpha - 1) = 1650 \times 81(1.20 \times 0.342 - 1) = -78{,}900 \text{ ft-lb}$$

$$T_s = wR^2(U \sin \alpha - \alpha) = 1650 \times 81(1.20 \times 0.940 - 1.222) = -12{,}600 \text{ ft-lb}$$

$$V_s = wR\alpha = 1650 \times 9.0 \times 1.222 = 18{,}150 \text{ lb}$$

These values for M, T, and V, together with other values calculated in like manner at several intermediate points along the slab, have been used to plot the curves shown in Fig. 3.

The author wishes to emphasize that the formula provided for midspan bending moment M_c and the curves of Fig. 1 are limited to the common case of uniform loading covering the entire span. Bending and twisting moments for concentrated loads, as well as uniform loads, on circular arcs are given directly by Michalos in Tables 3 and 4 of "Numerical Analysis of Frames with Curved Girders."[5]

Furthermore, it is not necessary to assume that the staircase is equivalent to its horizontal projection, although this assumption is believed by the author to be convenient and accurate enough for the practical analysis of moderately sized helicoidal staircases. However, it may be deemed advisable, in dealing with large and spectacular structures, to forego the simplifications offered by the author. For exact methods of analysis see references 6 and 7.

COMPUTATION OF TORSIONAL SHEARING STRESSES

The correct determination of torsional shearing stresses in a prismatic bar of noncircular cross section, subjected to twisting moments, was first achieved by Barré de St. Venant. His method was simplified by L. Prandtl, who in-

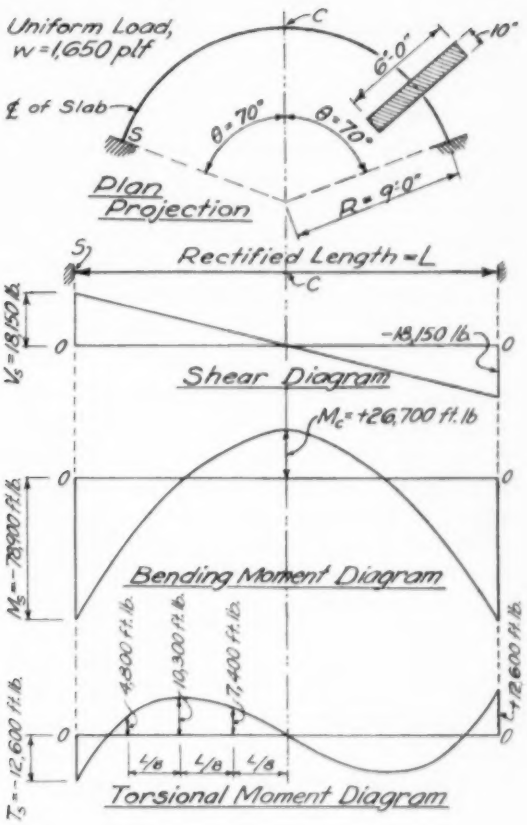

Fig. 3—V, M, and T diagrams for example

troduced a stress function, ϕ, appearing in the partial differential equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = -2G\theta$$

which combines in a single expression the conditions of equilibrium and compatibility. G is the modulus of shear, θ is the angle of twist per unit length along the bar, and x and y are the Cartesian coordinates of the cross section. The shearing stresses on the cross section in the x and y directions are, respectively:

$$\tau_{zx} = \frac{\partial\phi}{\partial y} \quad \text{and} \quad \tau_{zy} = -\frac{\partial\phi}{\partial x}$$

The stresses thus computed must, of course, be such as to satisfy the boundary conditions; *i.e.*, the correct solution for ϕ must satisfy the boundary conditions as well as the Poissonian equation. Ordinarily, the shearing stresses on the exposed faces, the bounding surfaces, of the prismatic bar are zero; hence, the value of ϕ along the boundary of a cross section of such a bar will be equal to an arbitrary constant which, for convenience, will be taken equal to zero (Fig.

4). Values of ϕ can be determined by the classical methods of the mathematical theory of elasticity (reference 8, chapter 11), or by relaxational methods (reference 8, p. 468; reference 9, section 5.6), or by experimental observations on soap films in connection with the membrane analogy. Some idea of the variation in ϕ and the distribution of shearing stresses over the cross section of a straight member subjected to torsion can be gained by a study of Fig. 4. Perhaps the most noteworthy feature is that the maximum torsional shearing stresses occur along the sides *closest* to the center of gravity of the section.

The maximum torsional shearing stress in a *straight* member of rectangular cross section, bh, can be computed with sufficient accuracy by

$$\tau_{max} = \frac{T}{bh^2}\left(3 + 1.8\frac{h}{b}\right) \qquad b \geq h$$

(in which T is the twisting moment), or with greater accuracy by use of data on pp. 277-278 of reference 8. This is the value of τ_{zz} which occurs at the centers of the long sides of the cross section. In straight members with cross-sectional ratio $b/h \geq 3$, the maximum torsional shearing stress paralleling the short sides is approximately equal to 75 percent of that along the long sides.

The discussion of shearing stresses so far has been confined to straight members. Actually, the stair slab is a sharply curved member, and shearing stresses computed by the foregoing equation intended for straight members are necessarily in error. The chief effect of the curvature is to increase the torsional shearing stress along the inner vertical side and decrease it along the outer. Göhner[10] has analyzed the effect of curvature on shearing stresses; some results of his analysis are incorporated in Fig. 5 which may be used to correct the shearing stresses calculated on the basis of a straight member. To the corrected torsional shearing stresses thus obtained there must be added *algebraically* the vertical shearing stresses due to the direct shear V. The direct shearing stresses can be calculated probably with minor error by the usual formula, $v = V/bjd$, although some argument might be presented for use of the expression, $v = 1.5V/bh$.

It should be mentioned that use of the calculated values of T and V without modification is somewhat erroneous. The computed values are the magnitudes of T and V in vertical planes, and not in the actual, inclined planes of normal cross sections. It is therefore permissible for the designer to reduce both T and V by multiplying by cos λ, where λ is the vertical angle between the horizontal and a tangent to the longitudinal centerline of the real, inclined slab.

DESIGN FOR SHEAR

The provision of adequate shear reinforcement is of critical importance in curved beams or slabs. Even where the calculated shear is zero, as at the center of span for a stair slab uniformly loaded throughout its length, it is highly advisable to provide some shear reinforcement, perhaps 50 percent of the maximum calculated anywhere along the span. Some designers provide the maximum calculated shear reinforcement undiminished throughout the entire span.

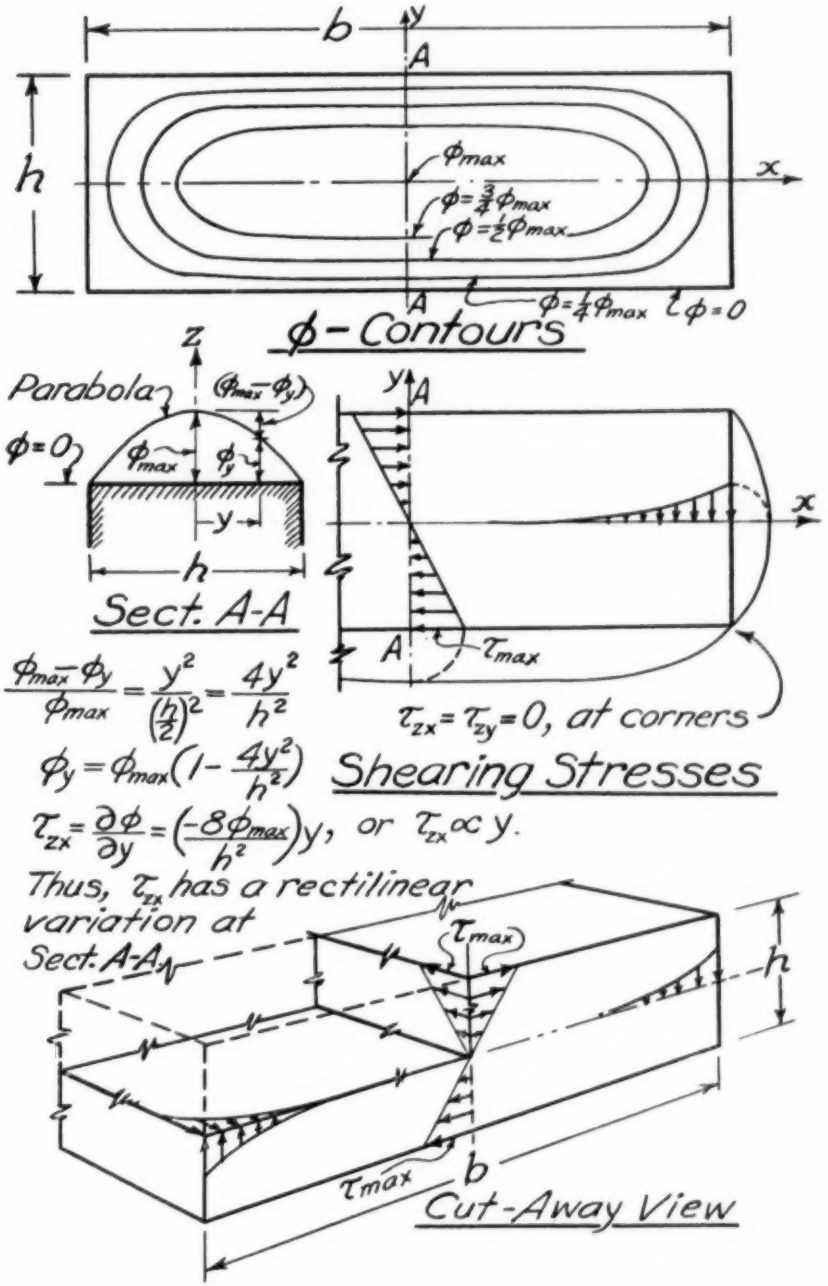

Fig. 4—Torsional shearing stresses

If a stair slab, or other member of rectangular cross section, is subjected to an excessive torque, failure will probably start by the opening of diagonal cracks appearing near the middle of the long sides, and making an angle of approximately 45 deg with the longitudinal axis of the member. As with ordinary reinforced concrete beams, such cracks are the result of diagonal tension.

The logical approach to the subject of reinforcing against torsion has been clearly and succinctly stated by Grinter[11] as follows:

> "Since vertical stirrups and horizontal rods have been found quite adequate for diagonal tension resulting from shear and flexure, there would seem to be no necessity for changing this proved and simple device when torsion is present. Failure is still by diagonal tension. The design criterion should still be to space hoops of any type in such a manner that every crack must cross at least one shear bar which itself is adequately anchored."

LONGITUDINAL REINFORCEMENT

No special or unusual procedure is necessary in calculating bending moment stresses. However, the longitudinal tension steel required for bending must be proportioned to carry also the longitudinal tensile component of the diagonal tension due to torsion; the hoop steel carries only the transverse component.[12]

Some of both the top and bottom bars should be made continuous throughout the full length of the span. For small or moderately sized stair slabs, the designer may well choose to simplify construction by calling for all longitudinal bars to extend full length; for large stair slabs, the designer may elect to save some steel by cutting off some (up to perhaps 50 percent) of the bars where his calculations indicate that they are not needed.

DESIGN AT THE SUPPORTS

Stair slabs are usually cast after the floors have been constructed. The necessary construction joints, together with the abrupt, gross enlargement of the slab at each support, must obviously invalidate any attempts to evaluate shearing stresses at the ends by St. Venant's theory. However, according to a helpful principle (also due to St. Venant), the particular distribution of stresses at the ends should not materially affect the calculation of stresses by St. Venant's theory at some distance, roughly equal to the slab width b, away from the ends.

For want of anything better, it is suggested that the designer deal with torque and direct shear at the supports by means of the familiar expression

$$p = \frac{V}{b} \pm \frac{6T}{b^2}$$

This equation indicates that there will be increased compression at one end of the slab seat, and uplift at the other. This uplift must be controlled by the addition of tie-down or anchor bars. As a safeguard, it is advisable to provide anchor bars at both ends of each slab seat, or even along the entire length of each seat.

Dowels, well anchored into the floor construction, must be provided to lap with

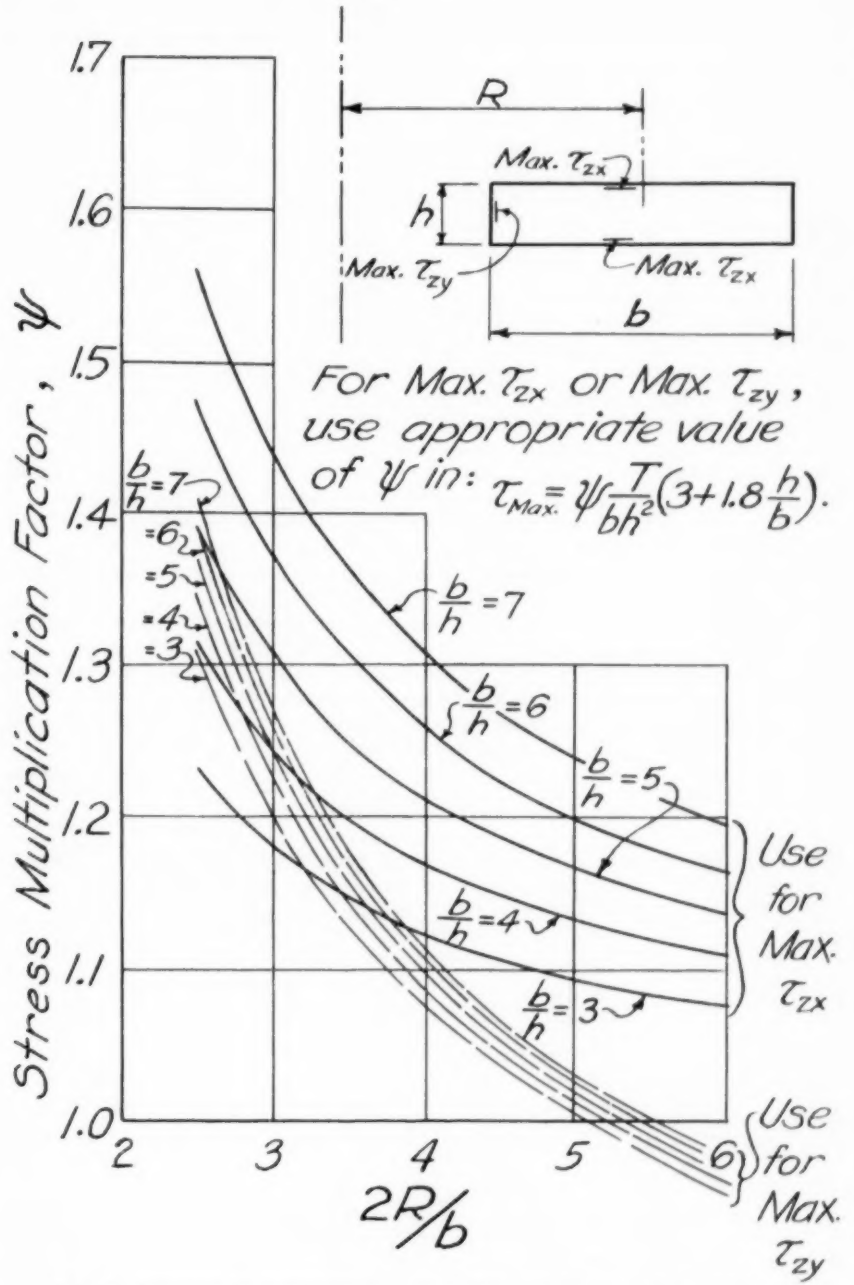

Fig. 5—Increase in maximum torsional shearing stress due to curvature

the longitudinal bars of the stair slab, if a simple extension of the floor slab steel is insufficient. Finally, it is essential to check both the strength and rigidity of the supporting members to ensure that the end fixity envisioned in the analysis will be realized in the structure.

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For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

Title No. 53-23

Fatigue Tests of Pre-Tensioned Prestressed Beams*

By A. M. OZELL† and E. ARDAMAN‡

SYNOPSIS

Fatigue tests were conducted to determine the behavior of pre-tensioned concrete beams prestressed with 7/16-in. seven-wire strands. Eight beams, 6 in. x 8 in. x 20 ft, with a center load, were tested to obtain some solutions to the following questions: Is the use of 7/16-in. seven-wire strand feasible from the standpoint of repetitive loads? Does a beam pre-tensioned with 7/16-in. strand fail in fatigue because of bond failure in the concrete or by breaking in the strand? What is the magnitude of changes in the load-deflection characteristics influenced by fatigue damage? What amounts of permanent set does a beam undergo when subjected to repetitive loads?

Pilot test results indicated that: (1) the use of 7/16-in. seven-wire strands is feasible; (2) the failure of the beams was due to the breaking of wires in the strands by fatigue; (3) load-deflection relationship was affected slightly during the first 30,000 cycles but greatly just prior to failure of the beam; (4) the permanent set was also small at first but increased appreciably just before failure. Results further indicated that no shear cracks were developed near the supports during the fatigue tests. It was possible to draw a load versus failure cycle diagram from the data obtained in these tests.

INTRODUCTION

The behavior of prestressed concrete beams subjected to repetitive loading has received little attention among engineers. Varieties of stressing units and strand sizes have been and are being used without much concern about their fatigue behavior. Further research needs to be undertaken so that results of a few tests carried out in Europe and in this country need not be dangerously extrapolated to cover all types, sizes, and designs of such units as they become available to the producers and users of prestressed concrete structural members.

The *Criteria for Prestressed Concrete Bridges*[1] specify that maximum strand size of seven-wire strands be limited to ⅜ in. To study the feasibility of using 7/16-in. strands from the standpoint of repetitive loads, a series of beams was

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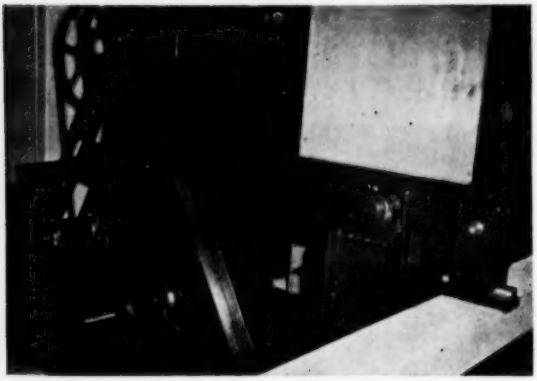

Fig. 1—General view of 10,000-lb capacity fatigue machine

tested in fatigue in which 7/16-in. strands were used as prestressing units.

In prestressed concrete beams excessive flexural loads produce tensile cracks in the concrete. These cracks presumably cause severe stress concentrations in the concrete and in the strands contiguous to the cracks rendering them vulnerable to fatigue. Tests were conducted to determine which of these stress concentrations ultimately caused the fatigue failure of the beam. In the tests a study was also made to determine the magnitude of changes in the load-deflection characteristics of each beam and of permanent set resulting from fatigue damage done to the beam.

DESCRIPTION OF THE FATIGUE MACHINE

The machine used for the repetitive loading of beams in flexure was designed and built in the testing laboratory of the civil engineering department of the University of Florida (Fig. 1). It is a constant-deflection type machine consisting of a rocker arm connected to an adjustable eccentric and speed-reducing belt-driven pulleys and is operated by an electric motor. Rate of load application is 140 repetitions per min. Deflections can be adjusted from 0 to 1.75 in. in magnitude.

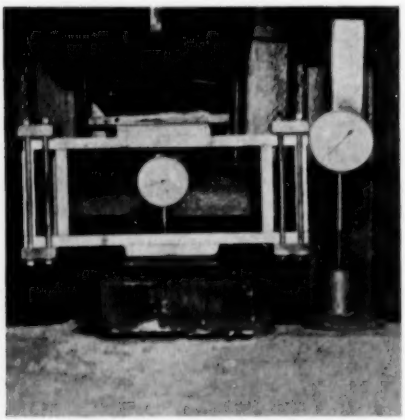

Fig. 2—Dynamometer used in beam calibrations. Permanent set in beams was determined by the Ames dial, right

Prior to the testing of a beam its initial load-deflection characteristic must be established by a static test since the load to be applied is determined from the corresponding deflection. For example, if a beam deflects 0.6 in. under a load of 2000 lb, then applying 0 to 0.6-in. deflection cycle to the beam would

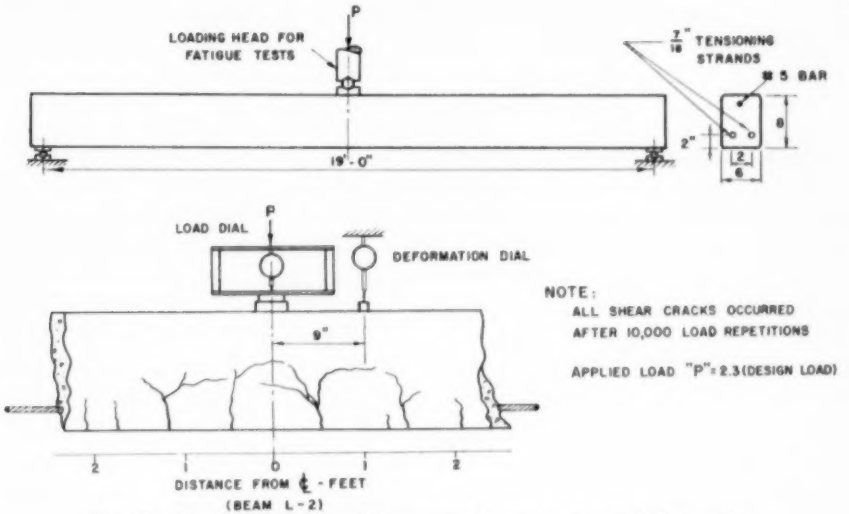

Fig. 3—General details of beam specimen and crack formation

be equivalent to repeating 0 to 2000-lb load for each cycle. However, when the load-deflection relationship of the beam changes after a number of cycles, the beam is recalibrated and the deflection adjusted accordingly to maintain a load of 2000 lb. This procedure is repeated until the beam fails.

TEST PROCEDURE

The initial load-deflection relationship of each beam was determined by loading the beam through a dynamometer as shown in Fig. 2. In every calibration each beam was loaded from zero load up to the test load so as to prevent excessive damage that would otherwise result from loads beyond this limit. To establish subsequent changes in the load-deflection characteristics of the specimen, the machine was stopped and the beam recalibrated at various intervals when such recalibrations were deemed necessary. If any appreciable (1 to 2 percent) change had occurred between any recalibration interval, the eccentric was readjusted so that the load on the beam remained almost constant during each test.

An Ames dial attached to the machine was used to measure the permanent deflection of the beam at each recalibration (Fig. 2).

DESCRIPTION OF SPECIMENS

Beam specimens subjected to repetitive loads were 6 x 8 in. in cross section and 19 ft center-to-center of end supports (Fig. 3). Prestressing was accomplished by two 7/16-in. seven-wire strands 2 in. from the bottom of the beam, and each strand was stressed to the initial load of 18,900 lb recommended by the manufacturer. Also, one unstressed #5 reinforcing bar, 19 ft long, was placed 1 in. from the top of the beam to reduce the compressive stress in the concrete and

prevent compression failure due to fatigue. General details of specimen and loading are given in Fig. 3.

Concrete used in the beam specimens had the following properties:

Volume proportions, dry rodded	1:1.7:3.5
Fine aggregate, average fineness modulus	2.19
Coarse aggregate, average fineness modulus	6.89
W/C ratio	0.627
Cement factor	6.5
Slump	3 in.

In these beams Type I cement was used with addition of ¼ oz of Darex and ¾ lb of Plastiment admixture per bag of cement, and the concrete was vibrated with a mechanical vibrator. Beams were made in groups of four, cast in one continuous line on two different days, each group designated here as series L and M. Four 6 x 12-in. cylinders were prepared from each batch. Some cylinders were tested after 5 days of moist curing at the time tension was released in the strands, and others after 13, 28, and 40 days dry curing. The specimens were cast by Florida Prestressed Concrete Co. of Tampa.

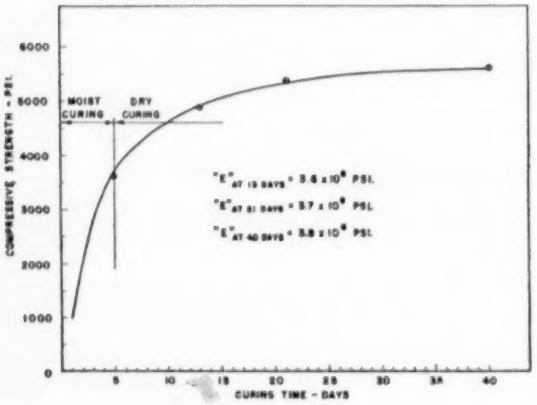

Fig. 4—Compressive strength and *E* of cylinders at various curing intervals

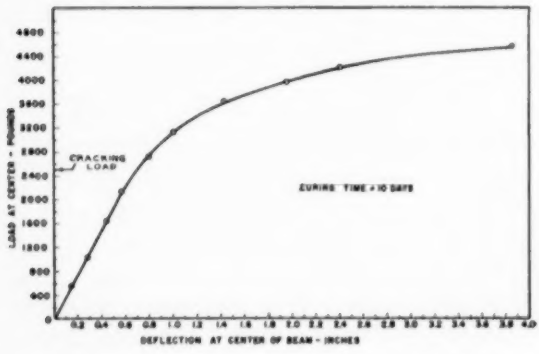

Fig. 5—Load-deflection relationship of beam L-1 up to ultimate load

TEST RESULTS

Static test results

The ultimate strength and modulus of elasticity of the cylinders at various curing intervals are given in Fig. 4.

Beam L-1 was tested statically to failure to determine its load-deflection characteristics up to the ultimate load. Fig. 5 shows the results of this test. It is interesting to note that the ultimate load for L-1 was 4580 lb, which is quite close to the theoretical ultimate load of 4620 lb calculated by using the expression $m_u = 0.9\, a_s f_s' d$ given in *Criteria for Prestressed Concrete Bridges.*[1]

Fatigue test results

A summary of test results is given in Table 1.

Beam L-2 was subjected to a repetitive load of 0 to 3160 lb, approximately 2.3 times the design load. The design load was taken as 1390 lb, corresponding to zero tension of the bottom fiber assuming 18 percent stress loss in the strands. The load-deflection characteristics (Fig. 6) showed slight variation up to 250,000 repetitions.

TABLE 1—SUMMARY OF TEST RESULTS

Beam No.	Center load, lb	Test load ÷ Design load	Number of cycles	Spring constant,* per lb in.	Loss in spring constant, percent	Permanent set, in.	Adjusted cycles of failure
L-1	4580	3.30	0	3680			Static test
L-2	3160	2.28	0	4250	0	0.02	460,000
			5,000	3800	10	0.03	
			50,000	3610	15	0.11	
			100,000	3130	26	0.12	
			250,000	2910	32	0.15	
			500,000	2500	41	0.23	
L-3	1360	0.98	0	4950	0	0	Did not fail
	1360	0.98	1,000,000	4170	16	0.03	
	1600	1.15	2,000,000	3890	21	0.07	
	2000	1.43	3,000,000	3650	26	0.09	
L-4	2500	1.80	0	5110	0	0	Did not fail
			5,000	4940	3	0.02	
			124,000	4920	4	0.05	
			2,000,000	4900	4	0.10	
			3,217,000	4660	8	0.14	
M-1	3420	2.46	0	5150	0	0.03	280,000
			94,000	4750	8	0.16	
			291,000	2840	45	0.23	
M-2	3235	2.33	0	5150	0	0.05	325,000
			152,000	3980	23	0.13	
			325,000	3200	38	0.28	
M-3	2730	1.96	0	5150	0	0.03	940,000
			5,000	4460	13	0.05	
			38,000	4120	20	0.10	
			245,000	4000	22	0.12	
			436,000	3670	29	0.16	
			823,000	3650	29	0.22	
			960,000	1940	63	0.31	
M-4	3800	2.74	0	5250	0	0.08	126,000
			25,000	4300	18	0.18	
			50,000	4200	20	0.19	
			100,000	4200	20	0.21	
			126,000	3750	29	0.32	

*Spring constant = $\dfrac{\text{Design load of 1390 lb}}{\text{Center deflection caused by 1390-lb load}}$

Note: The observed cracking load in all the beams was about 2500 lb.

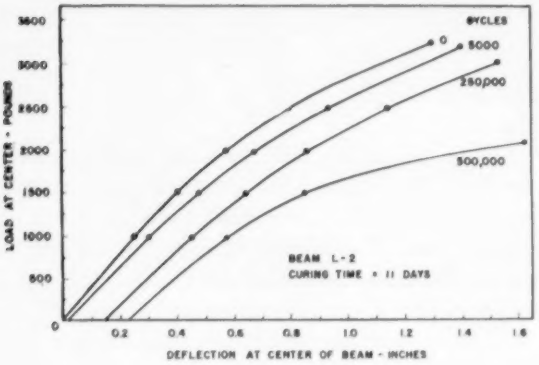

Fig. 6—Change in the load-deflection characteristics of beam L-2 at various repetitions of load

However, between the 250,000- and the 500,000-cycle calibrations the beam was badly damaged as evidenced by the considerable change recorded at the 500,000-cycle calibration, and this behavior was taken to signify failure. As will be discussed later, such abrupt variation was found to be caused by the breaking of the strands. On subsequent tests such a marked change in the load-deflection relationship of the specimen was taken to indicate failure.

In beam L-3, the test load was set at 1360 lb, and applied to the beam 1,000,000 times. Then the load was increased to 1600 lb causing a calculated 200 psi tension in the bottom fiber and this (second run) was also applied 1,000,000 times. The last load was set at 2000 lb causing a calculated 500 psi tension in the bottom fiber and this also (third run) was applied 1,000,000 times.

There were no visible cracks at the first application of the 1360-lb load or at the end of 1,000,000 cycles of repetition of this load. The same was true for the 1600-lb load. However, when the 2000-lb load was applied for the first time, three hair cracks appeared at the bottom of the beam under the load. The beam withstood the above cyclic loading without any appreciable change in its load-deflection characteristics.

Beam L-4 was subjected to a repetitive load of 2500 lb, which corresponds to 1.8 times the design load and 55 percent of the theoretical ultimate capacity. Load-deflection readings indicated that little damage was caused by the 3,200,000 repetitions of this load. Moreover, several small hair cracks, which did not progress appreciably during the test, were present from the first application of the load. The test was discontinued after 3,200,000 cycles at which time a permanent set of 0.14 in. was observed.

Beam M-1 was tested under a load of 3420 lb. After 94,000 load applications the permanent set was observed to be 0.16 in. The load-deflection characteristics of the beam remained substantially the same up to 291,000 cycles. At this point an appreciable change signified failure. The permanent set at 291,000 repetitions of load was 0.23 in.

Beam M-2 was tested at a 3235-lb load and failed at 325,000 cycles. For beam M-3, the test load was 2730 lb, approximately 2 times the design load, and failure

TABLE 2—STRESS RANGES IN THE BONDED STRAND

Specimen No. (1)	Test load, lb (2)	Calculated maximum strand stress, psi (3)	Stress range,* psi (4)	Corrected cycles at failure (5)	Column (4) ÷ +50,000,† percent (6)
L-4	2500	147,000	+5,000	did not fail	10
M-3	2730	160,000	+18,000	940,000	36
L-2	2160	183,000	+41,000	460,000	82
M-2	3235	189,000	+47,000	325,000	94
M-1	3420	202,000	+60,000	280,000	120
M-4	3800	222,000‡	+80,000	126,000	160

*Stress range in the bonded strand based on the calculated minimum stress of 142,000 psi.
†Allowable stress range in the unbonded strand.
‡Corresponds to about 0.05 percent permanent set in the strand.

occurred at 960,000 cycles. The change in the load-deflection characteristics was similar to those obtained on other beams. Slight variation was observed during the first 5000 cycles; larger variation occurred at the 500,000 reading. The change between the 823,000-cycle and the 960,000-cycle readings was significant, indicating failure.

Beam M-4 was tested at a 3800-lb load and failed at 126,000 cycles. There was an appreciable change in the load-deflection characteristics during the first 25,000 cycles which remained almost constant through the 100,000-cycle calibration. However, a considerable and sudden change was observed at the 126,000-cycle calibration. The permanent set, after the first application of load, was 0.08 in., the maximum for all beams tested in fatigue. The calculated strand stress in this beam was 222,000 psi (see Table 2) which corresponds to about 0.05 percent permanent set in the strand—probably the cause of the relatively high permanent set in the beam.

DISCUSSION OF BEAM FAILURES

Beam specimens stressed beyond the cracking load showed slight change in their load-deflection characteristics and underwent a small amount of permanent set (0.10 to 0.18 in. depending on the test load) during the first 30,000 cycles. However, the continued repetitions of loading caused considerable change at greater number of cycles. Also, the hairline cracks, which appeared at the first cycle, widened and progressed toward the middepth of the beam after 5000 to 10,000 cycles. At higher cycles these vertical tensile cracks changed direction, paralleling horizontal shear planes, indicating the cumulative tensile and shear damage in concrete as the cycles progressed (Fig. 3).

After failure, the beams were broken for visual inspection of the strands. This disclosed significant results. The wires were broken at points adjacent to the cracks near the center of the span. In one instance (beam M-1) as many as six wires of a strand were broken at one point and one other 9 in. away. In the beams that failed in fatigue at least three wires of one strand were broken at the same location.

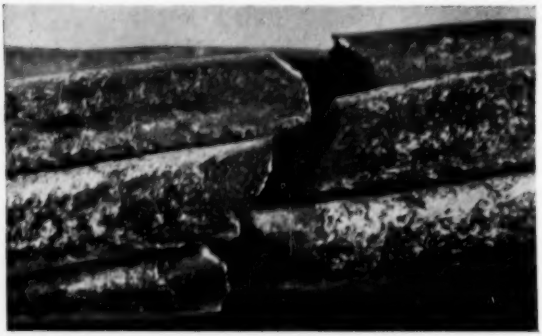

Fig. 7—Strand of beam M-1 after failure

The fracture of the wires occurring at points bordering the tension cracks in the beam substantiates the belief that these cracks form stress concentrations in the strands, especially as a result of overloads, rendering them vulnerable to fatigue and ultimately causing the breaking of the wires (see Fig. 7).

Except for one strand in beam M-1, there was no visual evidence of bond failure when the beams were broken into for study. Each strand had a coating of concrete particles along its length indicating no rubbing action which would have occurred had there been slip present. The strand in beam M-1, in which six wires were broken, showed definite signs of slip as the wires were smooth for a distance of about 1½ in. from the point of fracture.

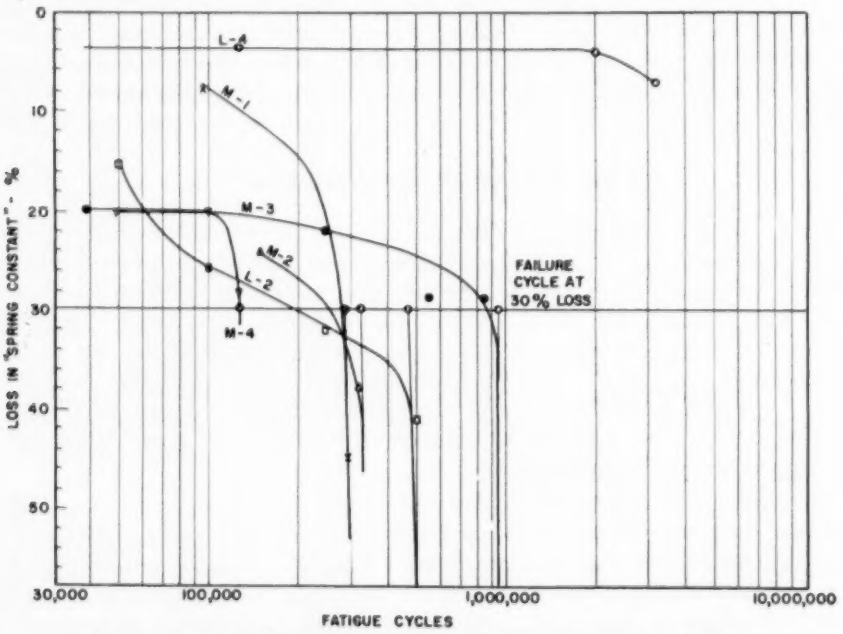

Fig. 8—Loss in the spring constant of beams caused by fatigue

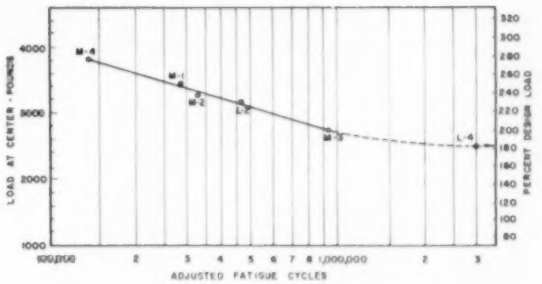

Fig. 9—Load versus adjusted cycles of failure diagram

It also should be noted that in the beams tested no shear cracks developed near the supports. However, further tests with loading conditions producing higher total shears upon which bond stresses are based should be conducted to study the behavior of 7/16-in. strands subjected to repeated loads.

DISCUSSION OF FATIGUE TEST RESULTS

Establishing the cyclic loading which caused initial failure in the strands was difficult, if not impossible. Unfortunately, the tests had to be continued until an obvious change had taken place in the load-deflection characteristics of the beam and this probably occurred after the fracture of several of the wires. Therefore, the authors chose to define fatigue failure as the cyclic loading that caused 30 percent loss in the "spring constant." Spring constant is defined here as the design load divided by the corresponding center deflection. Fig. 8 shows the percent loss in the spring constant for various beams at different cycles. An appreciable change in the curve signified the failure of the beam and was helpful in the graphical interpolation and "back-tracking" to the probable failure cycles which here will be called the adjusted cycles of failure.

The load versus adjusted cycles of failure diagram of Fig. 9 was drawn using the corrected cycle of failure and the test load for each beam.

Fig. 10 shows the relationship between permanent set of beams at various cycles of load repetitions. The permanent set was small at first and changed slightly until just prior to failure at which time it increased rapidly. It is interesting to note the similarity between the rapid changes in the permanent set and the spring constant curves of the beam just before failure. For overloads of more than twice the design load it was found that the permanent set just prior to failure was approximately twice that of the 10,000-cycle value.

Calculations were made to determine the additional stress in the bonded strands caused by fatigue loads. It was assumed that the cracks due to overloads extended to middepth, 4 in. from the bottom of the beam. (By actual measurement these were from 3.5 to 4.5 in.) It was assumed further that the stress distribution in the compression area was parabolic, giving a lever arm for the resisting moment couple of 4.5 in. or ¾ of the depth. The observed cracking load in all the beams was approximately 2500 lb, corresponding to a calculated strand stress of 147,000 psi. It should be noted that, assuming an 18 percent stress loss

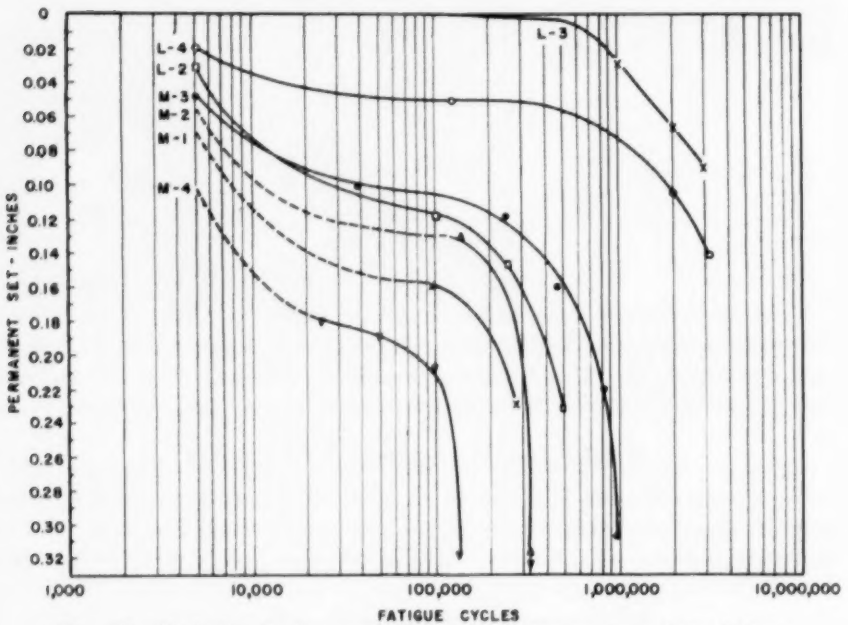

Fig. 10—Variation in the permanent set of beams caused by fatigue

had occurred up to the time of tests (1-3 months after stress release), the stress in the strand with no load was approximately 142,000 psi. With an assumed tensile stress in concrete of 600 psi at the level of the strand the corresponding stress would be about 4000 psi. Hence, at cracking load the total stress in the bonded strand would be approximately 146,000 psi which agrees with the calculations mentioned above.

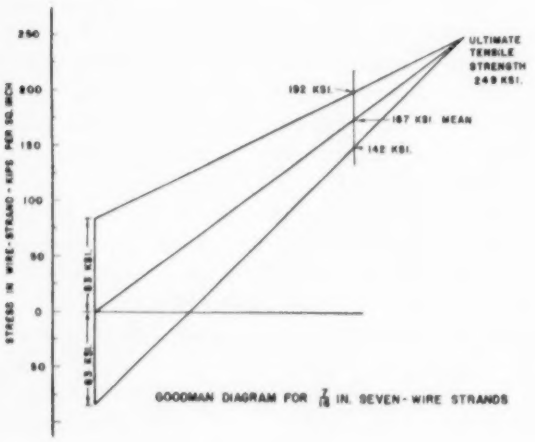

Fig. 11—Goodman diagram for minimum stress and maximum allowable range

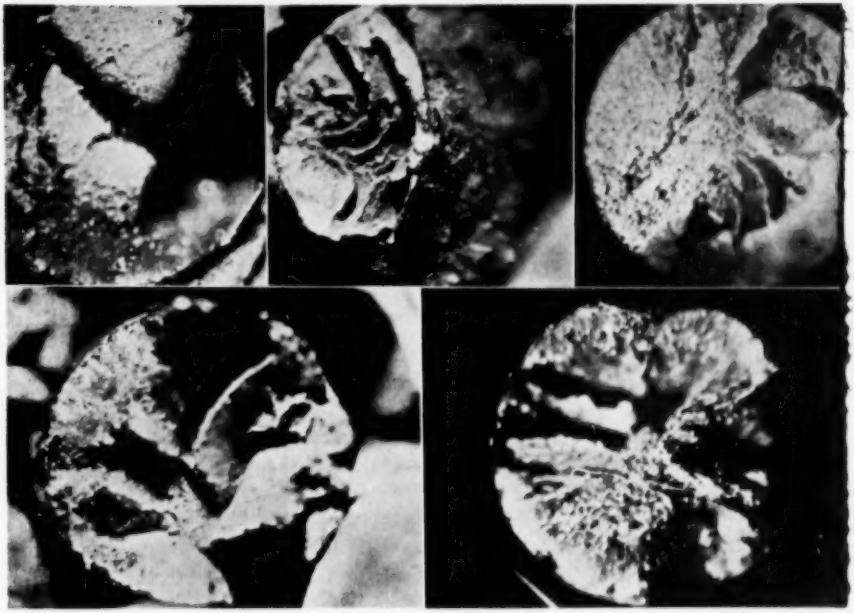

Fig. 12—Microscopic view of wire fracture. Note the fatigue type of fracture of the top three wires

The Goodman diagram[2] was drawn using the data given in the manufacturer's catalog and is shown in Fig. 11. The Goodman diagram is a graphical representation which gives an approximate maximum allowable stress range for a given minimum stress that a member can endure without failure when subjected to infinite number of repetitions of that particular stress amplitude. The diagram indicated that, based on a minimum stress of 142,000 psi, the unbonded strands have an allowable maximum stress of about 192,000 psi or a range of ± 50,000 psi. In the beam tested the calculated minimum stress was 142,000 psi, and the calculated maximum stresses varied as shown in Table 2.

Column six of Table 2 gives the ratio of stress range in the bonded strands to the allowable range of the unbonded strands. This ratio varied from 10 percent for beam L-4, which did not fail after 3,200,000 cycles, to 160 percent for beam M-4, which failed at 126,000 cycles. In beam M-2 the stress range in the bonded strand was 6 percent less than the allowable stress range of the unbonded strand, and failure occurred at 325,000 cycles, indicating the severity of the "stress-raiser" caused by the cracks in the concrete on the bonded strands. Supplementary tests should be conducted to verify these results.

Some metallurgical studies were undertaken also, to investigate the type of fractures that occurred in the wires. Micro-hardness tests were conducted on the broken wires taken from strands of beam M-2. Although there were slight increases in the hardness of the wires near the fractures, the values did not prove

to be conclusive inasmuch as the high carbon cold-drawn wires used in the manufacture of these strands are extremely high in hardness to begin with and do not clearly reveal the usual marked changes, as in the case of structural steel.

The broken wires taken from strands of beam M-3 were also studied under a microscope. As seen in Fig. 12 some wires exhibited fatigue type of fractures and others did not. This may be due to the fact that when one or two wires break in fatigue, much higher loads are transferred to the others causing them to fail statically. It is believed, however, that the crack, which develops in the first wire due to fatigue, progressively reduces the cross-sectional area enough to cause the rest of the wire to fail statically. This can well happen to several of the wires simultaneously. It would be desirable to establish with additional tests this process of progressive failure that seems to take place in the wires of bonded prestressing steel.

CONCLUSIONS

The following conclusions can be drawn from the results of pilot tests described above:

1. The use of 7/16-in. seven-wire strands in pre-tensioned beams of similar dimensions and the same loading conditions as in these tests seems feasible.

2. The flexural fatigue strength of the prestressed beams tested was approximately 1.8 times the design load. Overloads above this ratio had damaging effect on the fatigue life of such beams, *i.e.*, a load 2 times the design load caused strand failure at 940,000 cycles.

3. Fatigue failures were caused by the breaking of the wires in the strand and not as a result of bond failure. However, other loading conditions causing higher shears may induce bond failures.

4. The load-deflection relationship and the permanent set of the beams changed slightly during the first 30,000 cycles but extensively just prior to the failure.

REFERENCES

1. *Criteria for Prestressed Concrete Bridges*, Bureau of Public Roads, U. S. Department of Commerce, Washington, D. C., 1954.

2. *Prevention of the Failure of Metals Under Repeated Stress*, Bureau of Aeronautics, Battelle Memorial Institute, John Wiley & Sons, New York, p. 131.

3. *Roebling Tensioning Materials for Prestressed Concrete*, Catalog PC-929, John A. Roebling & Sons, Trenton, N. J., 1954.

For such discussion of this paper as may develop please see Part 2, December 1957 *Journal*. In *Proceedings* V. 53 discussion immediately follows the June 1957 *Journal* pages.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Construction

Factory-made prestressed concrete girders trucked 90 miles to site

D. P. Billington and M. E. Warner, *Civil Engineering*, V. 26, No. 8, Aug. 1956, pp. 34-36

Columns were eliminated from the floor area of cement plant building through the use of 75-ft span prestressed girders. Cost comparisons were made for the girders used, structural steel trusses, thin shell concrete, and structural steel rigid frames. The fireproof concrete roof adopted was slightly more expensive than the structural steel truss with metal roof deck.

Concrete slabs are laid for bridge abutment riprap

Contractors and Engineers, V. 53, No. 2, Feb. 1956, pp. 138-140

Wire-reinforced, 5-in. thick slabs were used to give erosion protection to earth fills on an overflow structure and a main river span. Strips were placed in alternate 4- to 5-ft widths, then intermediate strips were placed without forms. A ¾-in. slump concrete was used.

Forms on wheels speed warehouse construction

Contractors and Engineers, V. 53, No. 7, July 1956, pp. 26-30

Describes several shortcuts used in construction of a warehouse. Form panels for the structure made up into modular units of 12 x 23 ft, to fit the standard column spacing of the structure, were mounted on tubular steel scaffolding units. For each re-use, rollers were attached to the legs of the scaffolding and the entire section rolled into another position on the floor slab. The top surface consisted of joist bearer members and the forming operation was completed by installing standard steel pans. Other shortcuts employed included the use of pumps for placing concrete, drilled pile footings immediately below each column, and patented wall form units for the concrete walls.

Factory fabrication of dwellings (La fabrication en usine des batiments d'habitation)

Edouard Fougea, *Memoires*, Societe des Ingenieurs Civils de France (Paris), V. 108, No. 6, Nov.-Dec. 1955, pp. 445-464

Reviewed by Aron L. Mirsky

Fabrication and erection of concrete apartment houses. Walls, stairs, and other structural elements were factory prefabricated. Erection was by gantry bridges straddling the structure.

Reinforced concrete industrial buildings

The Indian Concrete Journal (Bombay), V. 30, No. 3, Mar. 1956, pp. 72-78, 96

Describes the application of flat slabs, beam and girder, and two-way slab to industrial buildings. Surprisingly notes

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A part of copyrighted Journal of the American Concrete Institute, V. 28, No. 4, Oct. 1956, *Proceedings* V. 53. Address 18263 W. McNichols Rd., Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books reviewed are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the "Current Reviews" section are available from ACI headquarters on request.

that the use of flat slabs for multistoried factories in India is not as popular as the other methods. An interesting argument for the use of a diagonal column layout is presented. Briefly describes the common types of thin shell roofs for single-story industrial buildings requiring large clear areas. Both butterfly and barrel shell roofs have been used in India.

Concreting bridges and cableways for the dams of the upper reaches of the Glockner-Kaprun Tauern power station (Betonierbruecken und Kabelkranfahrbahnen bei den Sperren der Oberstufe des Tauernkraftwerkes Glockner-Kaprun)

FRIEDRICH MESCHAN, *Der Bauingenieur* (Berlin), V. 30, No. 5, May 1955, pp. 175-185

Reviewed by ARON L. MIRSKY

Detailed descriptions of the concrete placing plant used in this large project. See also *Der Bauingenieur*, Jan. and Mar. 1954 ("Current Reviews," ACI JOURNAL, May 1955, *Proc.* V. 51, p. 952).

Dams

Effect of uplift on gravity-dam profiles

ERIC H. BROWN, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 5, No. 1, Apr. 1956, pp. 196-212

Reviewed by ARON L. MIRSKY

Discusses effect of internal hydraulic uplift on stresses in and profile of solid gravity dams. Also discussed is effect of internal drain arrangement on the distribution of vertical stress in concrete. Two design philosophies are considered and results compared: (1) no tension, with reservoir empty; (2) a small amount of tension cracking is permitted, hence upstream face need not be battered. Discussion is confined to dams of medium height (400-500 ft using 500 psi as maximum permissible concrete stress).

Design

Moments in flat slabs

M. W. HUGGINS and W. L. LIN, *Proceedings*, ASCE, V. 82, ST 4, July 1956, pp. 1020-1 to 1020-30

Summarizes the results of a study of moments in a cast aluminum model of a flat slab floor. Round capitals and columns were used. There were no drop panels. Results are compared to those obtained using the continuous frame analysis method recommended in the ACI Building Code (ACI 318-51). A comparison of the test results and the analyses indicates: (1) Test moments for positive bending at midspan were higher in all cases than those by the recommended method of analysis. (2) The *maximum* negative test moment was always substantially higher than the corresponding *average* negative moment by the analysis. (3) Maximum negative moments in middle strips were approximately equal to the moments found by the analysis except that they occurred on column centerlines rather than at the specified critical sections. The concluding recommendation consists of factors by which the positive moments from an analysis should be multiplied to determine the design moments. These factors range from 1.25 to 1.90 for live load moments and from 1.40 to 3.40 for dead load moments.

Bending stresses in edge-stiffened domes

M. G. SALVADORI and R. SHERMAN, *Proceedings*, ASCE, V. 82, ST 4, July 1956, pp. 1021-1 to 1021-28

Bending moments and shears in certain types of thin shells of revolution stiffened by an edge beam are obtained and graphs for the evaluation are given. Variable symmetrical live and dead loads are considered together with temperature and shrinkage stresses. Approximately determined but sufficiently accurate values for the maximum moments in the shell and supporting cylinder are derived by means of the same graphs

when the dome is built into a cylinder and stiffened by an edge beam. The graphs presented allow the quick completion of the analysis and design of a thin-shell dome structure by providing the designer with a solution for the bending stresses which may be combined with the "membrane" stresses for the remainder of the dome.

Influence lines for continuous beams. Part 3—Four span continuous beams, constant moment of inertia

WALTER C. BOYER and JOEL I. ABRAMS, *Technical Report* No. 7, Department of Civil Engineering, Johns Hopkins University, May 1956, 135 pp.

Influence data—reactions, support moments, and various maximum interior span moment combinations—in tabular form are presented for four-span continuous beams. All data are in dimensionless form. Contains data for 92 selected cases giving close coverage in the range of span ratios 1.43 to 0.7. A span ratio is defined as the second, third, or fourth span length divided by the first span length. Parts 1 and 2 covered two- and three-span continuous beams (see "Current Reviews," ACI JOURNAL, July 1956, *Proc.* V. 53, p. 136).

Load distribution in highway bridge decks

A. W. HENDRY and L. G. JAEGER, *Proceedings*, ASCE, V. 82, ST 4, July 1956, pp. 1023-1 to 1023-48

Presents a general method for calculating distribution of longitudinal moments, deflections, etc. in bridge decks. Develops a theory and compares results of tests on structures and models to justify its application. Rather good agreement is obtained between computed and measured values in these comparisons. It is noted that the main difficulty in applying the theory is not the distribution theory itself but in evaluating the effective moments of inertia, torsional constants, and elastic moduli of the bridge elements—particularly when they are composite concrete and steel members. It is shown, however, that even relatively wide variations in these assumed properties result in relatively small variations in the distribution coefficients.

Cracks in reinforced concrete (in Danish)

B. KUDSK-JORGENSEN, *Beton og Jernbeton* (Copenhagen), V. 7, No. 4, Oct. 1955, pp. 159-170

Reviewed by T. HEEMANSEN

A Swedish formula is introduced for the computation of the size of tension cracks in reinforced concrete members. Based on this expression the author has set up a design formula for reinforced concrete beams, which takes the crack width into account. Diagrams are given for rectangular sections and T-sections. Method is illustrated by examples.

Restraint of reinforced concrete slabs by torsionally rigid edge beams (in German)

H. LINDNER, *Beton- und Stahlbetonbau* (Berlin), V. 50, No. 10, Oct. 1955, pp. 264-267

APPLIED MECHANICS REVIEWS May 1956 (Mueller)

Author shows how the moments in the slabs can be calculated if the torsional rigidity of the edge members and the concentrated loads due to vehicles are included, provided certain simplifying assumptions are made.

Simplification of design by ultimate strength procedures

P. M. FERGUSON, *Proceedings*, ASCE, V. 82, ST 4, July 1956, pp. 1022-1 to 1022-25

Design procedures for beams and columns using the simple rectangular stress block are developed within the recommendations of the report of the ACI-ASCE joint committee on ultimate strength design (ACI JOURNAL, Jan. 1956, *Proc.* V. 52, p. 505). Includes illustrations of designs with or without the design aids (curves) available in the joint committee report. Particularly emphasizes the fact that design by the ultimate strength method employs much simpler design equations than the conventional elastic methods.

Design chart for the economic section for prestressed concrete beams

REGINALD G. ROBERTSON, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 5, No. 1, Apr. 1956, pp. 184-195

Reviewed by ARON L. MIRSKY

Develops equations and charts connecting section area, depth, and cable pull with allowable working load stress and live and dead load moments, for I-shaped sections (equal or unequal flanges).

Exact analysis of multistoried structures subjected to wind loads

B. N. THADANI, *The Indian Concrete Journal* (Bombay), V. 30, No. 3, Mar. 1956, pp. 87-91

Presents moment distribution solutions for side sway and wind loads on multistory rigid frames.

Structural failures and how to avoid them

JACOB FELD, *Journal*, Boston Society of Civil Engineers, V. 43, No. 2, Apr. 1956, pp. 108-127

Reviewed by ARON L. MIRSKY

Defining structural failure as the nonconformance of the structure with design expectations, author provides a brief but graphic lesson in what *not* to do (the topics of what to do and how to do it have been well covered, he says, in formal textbooks and courses). Engineers old and young could read this paper with profit.

Dynamic loads with special consideration of impact (in Serbian)

S. PUKL, *Naše Gradevinarstvo* (Belgrade), V. 10, 1956: No. 3, pp. 359-363; No. 4, pp. 505-509

Reviewed by J. J. POLIVKA

For structures under dynamic loads and impact, mass and time must be considered. Author discusses mathematical methods already known, and points out also that physical phenomena at the point of application of kinetic energy must be considered in calculating the balance of energy. Two cases are investigated with mathematical presentation: (1) cases in which nearly the whole kinetic energy due to the dynamic loading is transformed into deformations or oscillations of the structure; (2) cases in which only local deformations or failure at the point of application result. Equations for both cases are derived and criteria presented for speedy classification of the proper case and simplified solution.

General study of the elastic deformation of thin shells (Etude generale de la deformation elastique des voiles minces)

M. LAYRANGUES, *Annales des Ponts et Chaussees* (Paris), V. 126, No. 1, Jan.-Feb. 1956, pp. 39-76

Reviewed by ARON L. MIRSKY

Vectorial methods used in author's previous study of the static equilibrium of thin shells (*Annales des Ponts et Chaussees*, V. 123, No. 1, Jan.-Feb. 1953, pp. 69-105; "Current Reviews," ACI JOURNAL, June 1953, *Proc.* V. 49, p. 965) are here used in the presentation of a general theory of the elastic deformation of such structures. Method is then applied to shells whose median surface is a right cylinder or a surface of revolution.

Analysis of gable frames by the column analogy method

E. S. E. LYLE, *Concrete and Constructional Engineering* (London), V. 51, No. 6, June 1956, pp. 389-394

Presents a quick method of solution for the special problem of gable frames of one story and one bay. The derivation of the method is carried through for a general case followed by an example and for the special cases of symmetrical frames and frames with columns hinged at the base with numerical examples.

Materials

Osmotic studies and hypothesis concerning alkali-aggregate reaction

G. VERBECK and C. GRAMLICH, *Proceedings*, ASTM, V. 55, 1955, pp. 1110-1131 (including discussion)

Describes an osmotic cell technique for the study of the chemistry and physics of the alkali-aggregate reaction. Experimental data include the effect of (1) alkali concentration, (2) different alkalies, (3) calcium hydroxide concentra-

tion, (4) particle size of aggregate, and (5) reaction temperature as variables. The hypothesis developed is that the course of the reaction of the silica with alkali or calcium hydroxide proceeds on a competitive basis at the surface of the reactive aggregate and that the nature of the products, deleterious or innocuous, depends upon the relative availabilities at the silica surface of alkali and calcium hydroxide, respectively.

Accelerators of setting and hardening of cement (in Russian)

V. V. Kind, E. V. Lavrinovich, and R. E. Litvinova, *Tsement*, V. 21, No. 3, 1955, pp. 7-12
Ceramic Abstracts
May 1956 (Kamich)

One of the basic factors of the acceleration of setting of cement in the presence of additions (accelerators) is the increase in its specific surface caused by the dispersion of cement grains by the action of the additions. Reduction in setting time is also facilitated by an increase in solid phase owing to the formation of insoluble products of the reaction of lime with the additions of salts. During the hardening of portland cement mixed with $CaCl_2$ solutions the $CaCl_2$ is apparently combined, with the formation of difficultly soluble Ca oxychlorides and Ca chloraluminates.

Studies on the suitability of expanded shale aggregate for use in cement concrete

Dave H. Sawyer, Claude M. Brown, Jr., and Loran H. Strunk, *Bulletin* No. 38, University of Kentucky Engineering Experiment Station, Dec. 1955, 26 pp.
Highway Research Abstracts
Apr. 1956

In brief, the investigation has shown that concrete mixes containing expanded shale aggregate can have: (1) compressive strengths approximately equal to those of comparable concrete mixes containing dense aggregate; (2) flexural strengths slightly lower than those of comparable concrete mixes containing dense aggregate; (3) a satisfactory bond with reinforcing steel to the yield point of the steel, even though the relation between slip and bond stress is somewhat less favorable for the light-aggregate concrete than for comparable dense concrete; (4) an extremely poor resistance to freezing and thawing as it is carried out in the usual manner of durability testing; (5) an inherent resistance to resaturation after having been dried—a factor that could be of considerable value with regard to durability under some service conditions; (6) an increase in resistance to freezing and thawing through use of air entrainment; (7) a high resistance to abrasion—as measured by a modified Los Angeles type test; and (8) unit weights approximately 35 percent less than those for corresponding mixes containing limestone aggregate.

Big bars for bridges

R. C. Richardson, *Western Construction*, V. 31, No. 3, Mar. 1956, pp. 68, 72
Highway Research Abstracts
Apr. 1956

In late 1954 the California Department of Public Works consulted several major steel mills to determine the feasibility of rolling a round section with hi-bond characteristics which would be equivalent in cross-sectional area to the 2-in. square bar. After considerable study design sketches were submitted to the state providing for a bar with an elliptical cross section and an average diameter of 2⅜ in. The projection and spacing of deformations were determined by increasing the values in proportion to those for standard sizes established by ASTM specifications. The section was assigned bar designation #18.

The employment of 2⅜-in. diameter reinforcing bars in the design of major structures is mainly a matter of economy. With the 2⅜-in. diameter bars concrete columns can be designed to carry the same loads with substantial reduction in size. The larger steel area of the individual bars promotes simplicity and economy of beam and girder construction. This provides for less concrete usage, lower form cost, and can mean more usable space in the completed structure. Instead of laps, bars are joined by butt welding.

For the Eastshore Freeway now under construction in Oakland, Calif., 1720 tons of the #18 bars are required. Bars are lifted to placement elevation with cranes. They can then be placed individually or, when necessary, can be tied into cages on "column horses." The "column horse" is a manual hoisting (and bolstering) implement, consisting of pipe frame construction and hoisting tackle. By adding travelers to the tackle, cages can be conveyed manually to the area of emplacement. Bars up to 93 ft 6 in. in length will be used. At the site bars will be beveled and positioned for welding. The ends to be joined are preheated to 400 F. The thermal intensity is measured with a "tempilstik," which melts at this temperature. The bars are then butt-welded with low-hydrogen electrodes.

The #18 bar is now being used in refinery construction, schools, state highway bridges, overpasses, underpasses, separation, and distribution structures.

Pavements

Slip-form paver proves fast, economical road widener

RALPH MONSON, *Contractors and Engineers*, V. 53, No. 2, Feb. 1956, pp. 74-78

Slip-form pavers, capable of placing 10,000 linear ft of 3 ft wide concrete pavement per day have been used for road widening in Iowa. Cost ranges from $3.90 to $4.50 per sq yd. Equipment and construction procedure are described.

Suggested quality control method for highway concrete

G. MAXON, *Roads and Streets*, V. 99, No. 7, July 1956, pp. 72-78

Suggests three methods for field control of concrete uniformity intended to supplement and somewhat to replace the standard test methods which, presumably, would still be used for selecting the proportions and all preliminary tests. The three field-control methods suggested are: (1) a penetration test for consistency; (2) the use of a pocket-sized air meter for measuring air content of the mortar fraction in the concrete; and (3) a heavy-media method for determining the cement content in fresh concrete. The latter test would be used to indicate efficiency of the mixing by being performed upon samples from the first and last tenth of batches, twice a day. The air content and consistency tests would be made much more frequently than the usual slump test or present air content test.

Precast Concrete

Measurement of dimensional changes in precast concrete walls

F. A. BLAKEY and R. E. LEWIS, *Constructional Review* (Sydney), V. 29, No. 5, May 1956, pp. 23-27

Dimensional changes of the precast concrete walls were recorded over a period of 6 years. The walls were solid, 3 in. thick, 9 ft high, lengths up to 27 ft, and composed of 6-sack concrete. Total contractions up to 0.1 percent occurred and seasonal fluctuations of 0.05 percent. Findings of particular interest were that movement in interior walls is as great or greater than that of exterior walls, and that dimensional changes in the vertical direction are greater than those in the horizontal direction.

Prestressed Concrete

Prestressed cabled roof hangs from thin concrete stadium wall

Architectural Record, V. 120, No. 1, July 1956, p. 219

A concrete roof draped from the upper perimeter of a 4 in. thick concrete cylinder encloses a simple but functional arena in Montevideo, Uruguay. The stadium is 310 ft in diameter and 83 ft high, with no interior columns or supports. The roof system consists of 256 radial, high-tensile steel wire strands, sloping downward with a drop of about 1/10 the diameter to a central tension ring. The roof covering consists of about 9000 trapezoidal shaped, precast concrete slabs about 2 in. thick, fastened to the cables by hooks. After the slabs were

positioned, a temporary "prestressing" overload of bricks was applied and the joints between all slabs grouted. After the grout had set, the "prestressing" overload was removed. The 4 in. thick concrete cylinder walls were erected by the slip-form method.

Freight terminal of prestressed concrete (in Danish)

SOREN MOLDRUP, *Beton og Jernbeton* (Copenhagen), V. 8, No. 1, Jan. 1956, pp. 44-52
Reviewed by T. HERMANSEN

Describes a 17,000 sq ft, one-story freight terminal made of precast prestressed concrete elements. Roof girder is 66 ft long with cantilevers at both ends, 11 ft and 16 ft, respectively. Girder and cantilevers were fabricated separately of pre-tensioned concrete. Before erection the cantilevers were connected to the girder by post-tensioning using three curved Freyssinet cables, two of which were continuous.

Pre-tensioned piles are 132 ft long

Engineering News-Record, V. 157, No. 3, July 19, 1956, pp. 33-36

Describes the design, fabrication, and placing of pre-tensioned prestressed concrete piles used in constructing a San Francisco pier. The 2000 total prestressed concrete piles included 139 which were 131 ft 8 in. long. The long piles had a 20 in. octagonal section with an 11 in. diameter hollow core. They were pre-tensioned by eighteen ⅜-in. wire strands.

Prestressed concrete as applied to building frames

FRANCIS WALLEY and HUGH C. ADAMS, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 5, No. 1, Apr. 1956, pp. 70-104 (including discussion)
Reviewed by ARON L. MIRSKY

Brief paper describing development of use of prestressed concrete in buildings, from simply supported prestressed members through partly continuous to fully continuous ("framed") multibay, multistory structures. Typical calculations for a single portal frame with pinned ends, and notes on the design of an office building at Kilburn, London, are given in two appendixes.

Properties of Concrete

Lightweight concrete (in Danish)

B. SIMONSEN and P. WILCKEN, *Beton og Jernbeton*, (Copenhagen), V. 6, No. 4, July 1954, pp. 119-130
Reviewed by T. HERMANSEN

To determine the relation between strength and density of cement-bound lightweight concrete, tests were carried out at the Technical University of Denmark. Test results are reported for both normally cured and steam-cured concrete.

Basic considerations pertaining to freezing-and-thawing tests

T. C. POWERS, *Proceedings*, ASTM, V. 55, 1955, pp. 1132-1155

Questions current freezing-and-thawing tests and suggests an alternative test. The alternative test is preceded by a discussion of the mechanisms of frost action in cement paste and in aggregate. The proposed procedure would determine an "immunity period" to frost attack as the primary measure of frost resistance, this immunity period being the time required for the specimens to absorb enough moisture to show a dilation during freezing.

Studies of flexural strength of concrete. Part I—Effects of different gravels and cements

STANTON WALKER and D. L. BLOEM, *Joint Research Laboratory Publication* No. 3, National Sand and Gravel Association and National Ready-Mixed Concrete Association, July 1956, 24 pp., $0.50

Presents the results of tests to evaluate the effect of different coarse aggregates and cements upon flexural strength. Includes comparisons of flexural and compressive strengths, air entrainment, and cement factors. Concludes that aggregate properties affect flexural strength somewhat more than compressive strength but that fairly reliable relationships between flexural and compressive strengths for a given aggregate may be established, thus allowing the use of compressive strength tests for control. No correlation was pos-

sible between aggregate properties investigated and either compressive or flexural strength. Variation among cements in their flexural strength producing properties was small and was related to the variation in their compressive strength producing properties. Further studies are planned of the effects of other variables on flexural strength. The effect of surface roughness of the various coarse aggregates was not included in the scope of these studies.

Rapid-setting concrete (in Russian)

A. E. Desov, *Stroitelnaya Promyshlennost* (Moscow), No. 10, 1955, pp. 30-32. *Chemical Abstracts*, V. 50, No. 4, Feb. 25, 1956, p. 2945

Highway Research Abstracts May 1956

Effect of placing time was determined on 7 percent gypsum concrete made with cement beginning to set in 22 min. Placed directly, after 1 and after 1½ hr, it showed crushing strength after 24 hr of 232, 254, and 276 kg per sq cm, respectively, thus recommending a somewhat slower placing time. The water-cement ratio has a pronounced effect on this strength; increase thereof from 0.5 to 0.6 cuts the crushing strength in half. Addition of gypsum renders concrete fully low-temperature resistant but does not cause any corrosion. Gypsum causes a certain expansion of concrete, but the pattern of the latter is not a direct function of the concentration.

Structural Research

Determination of bending moments in plates with the aid of an optical mirror system (in German)

W. Koepcke, *Beton- und Stahlbetonbau* (Berlin), V. 50, No. 8, Aug. 1955, pp. 210-216

Applied Mechanics Reviews May 1956 (Gretter)

Optical (reflection) method is presented for determination of bending moments in plates. Model of specially heat-treated and ground plate glass is supported horizontally in a rigid framework and loaded from beneath by calibrated spring spindles or weighted levers. Camera mounted with aperture in plane of raster plate which is mounted above and parallel to model takes photographs of raster pattern as reflected in model surface. Raster pattern consists of rectangular array of mutually tangent small circles of equal diameter. Variations in slope of loaded plate lead to variable shifts of points in reflected image. Reflections of circles thus distort into shapes which are approximately elliptical. Stereo-comparison of photos of raster as reflected by loaded and unloaded plate gives data from which can be calculated principal curvatures and their directions. Brief discussion of use of similitude in extrapolating to prototype plate is included. Traveling telescope fitted with scales and small section of raster plate for determining of curvatures without photography is described.

General

Rational isolation of large precision machinery (L'isolation rationnelle des grandes machines de precision)

I. E. Katel, *Memoires*, Societe des Ingenieurs Civils de France (Paris), V. 109, No. 1, Jan.-Feb. 1956, pp. 23-33

Reviewed by Aron L. Mirsky

General discussion of vibration control in plants using rotating machinery such as newspaper presses, etc., by means of sheets of fluted natural cork placed between the upper and lower portions of the concrete foundations.

Overpressure of emptying in silos (Sobrepresión de vacindo en los silos)

Marcel Reimbert, *Informes de la Construcción* (Madrid), No. 78, Feb. 1956, Paper No. 582-5, 6 pp.

A study of variation of pressure in silos during the loading and unloading of grain. The object of the work, besides the general disposition and grouping of silos, was to determine the most appropriate means of avoiding overpressures while emptying the silo. Pressure curves from tests on full-size silos are shown.